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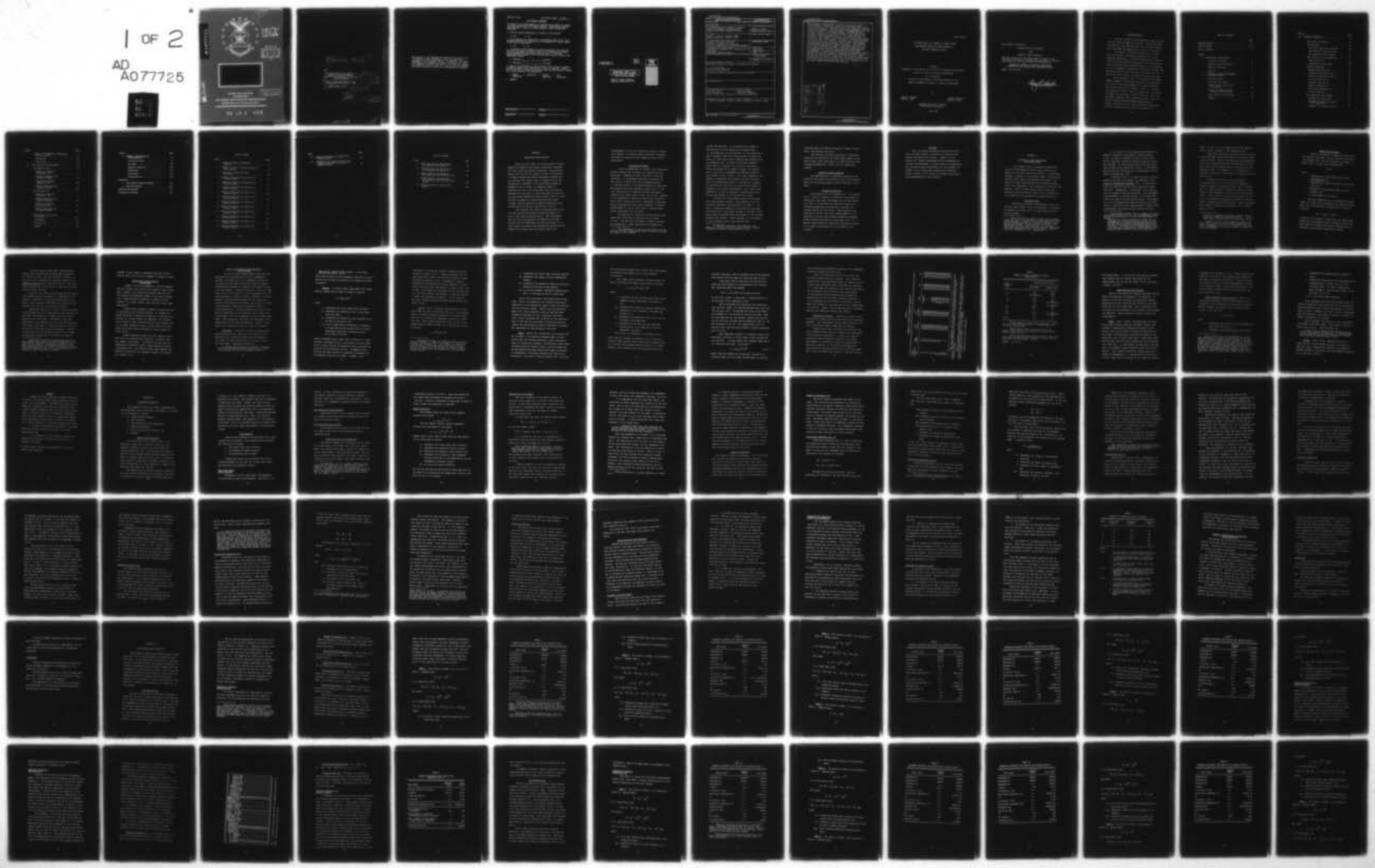

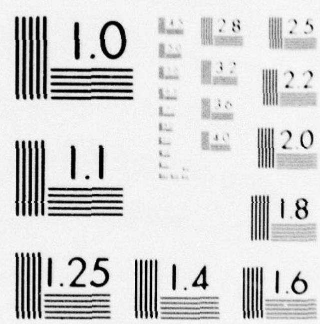
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Master's thesis

AN INVESTIGATION OF CHANGES IN
DIRECT LABOR REQUIREMENTS
RESULTING FROM CHANGES IN AVIONICS
PRODUCTION RATE.

David Y. Stevens, Captain, USAF
Jimmie Thomerson, Captain, USAF

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This research investigated the effects on direct labor requirements by exogenous changes in production rate in the ARC 164 radio and the Computer Signal Data Converter avionics production programs. The basis for the investigation was the 1975-1976 research by Larry L. Smith. He modified the standard learning curve model by adding a production rate variable using test data from the F-4, F-102, and KC-135 programs. ~~Smith found that~~ within the modified model, the production rate showed a significant and inverse relationship to direct labor requirements. Moreover, this modified model was more accurate in predicting direct labor requirements than was the standard learning curve model. This research extended Smith's modified model to avionics, validated it there, and confirmed its superior predictive ability, both statistically and subjectively. Therefore, Smith's model is recommended for use as a predictor of direct labor requirements in ongoing avionics production programs. Still another part of this research was modifying Smith's Fortran IV program, originated during his 1975-1976 research. The program was modified to include options for predictive ability tests and for projection sensitivity matrices. Finally, the modified interactive program has been made available in the Air Force COPPER IMPACT software library.

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AN INVESTIGATION OF CHANGES IN DIRECT LABOR REQUIREMENTS RESULTING FROM CHANGES IN AVIONICS PRODUCTION RATE

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

David Y. Stevens
Captain, USAF

Jimmie Thomerson
Captain, USAF

June 1979

This thesis, written by

Captain David Y. Stevens

and

Captain Jimmie Thomerson

has been accepted by the undersigned on behalf of the faculty of the School of Systems and Logistics in partial fulfillment of the requirements for the degree of

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Larry L. Smith
COMMITTEE CHAIRMAN

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CHAPTER I

INTRODUCTION AND OVERVIEW

Since the late 1960s, the United States Air Force acquisition community has become increasingly concerned with cost growth and resource allocation. The principal factors which led to this concern include the following: (1) weapon systems have become increasingly complex and the resultant cost growth tends to undermine public and Congressional confidence in the procurement process as managed by the Air Force; (2) competing needs for resources have led to the requirement that each need be assessed more accurately to allow equitable allocation of resources among defense and other projects; and, (3) there has been a tendency to reduce quantities when costs increase, thereby mandating purchasing techniques which forecast final prices more realistically (25:1-1).

This research will focus on developing more flexible cost and price estimates through improvement in direct labor cost-estimating models. Such improvements can help trim the commitment and expenditure of many dollars, and the improvements can help eliminate, or at least reduce, the public and Congressional perception that expenditures beyond those anticipated result from

mismanagement of Air Force acquisition dollars. Finally, this research, in aiming at better forecasts, addresses the effect of production rate changes on direct labor requirements.

Limiting the Problem

During the Department of Defense (DOD) acquisition process, complex cost estimates are required for many activities. Examples of these activities include the following: (1) procurement planning: to evaluate the cost likely to result from newly-proposed programs; (2) contract pricing: to establish negotiation objectives and determine fair and reasonable prices; and, (3) contract administering: to measure progress and compare planned versus actual costs to date (10:154-155). Because of the importance of obtaining accurate cost estimates within each of these activities, improvement in cost estimating techniques has been a major objective of PROJECT COPPER IMPACT II, the Air Force plan to improve modern pricing techniques (23:2,36,40-41).

One approach for production cost estimating used extensively within DOD is the learning curve model (10:465). The learning curve model is based on several key assumptions.[1] These assumptions or preconditions

[1]The assumptions of learning curve models and the questions by some investigators about linearity are beyond the scope of this research.

include the following: (1) the production process is characterized by a high proportion of manual labor; (2) the production is uninterrupted; (3) the production is that of complex items and is characterized by repetitive tasks; (4) there are no major technological changes during the production run; (5) there is continuous pressure and/or desire to improve; (6) carry-over knowledge is either not present or if present is taken into account; and (7) there are no exogenously introduced production rate changes (11:234; 15:135). This final assumption, the absence of production rate changes, has been the focal point for considerable research in attempting to improve the accuracy and flexibility of learning curve models in estimating direct labor requirements. This research was prompted by the lack of systematic consideration of production rate changes by the learning curve model.

A variation of the standard learning curve model which does consider changes in the production rate is a cumulative production and production rate model.[2] The use of a cumulative production and production rate model has resulted in a significant increase in accuracy of predicted costs for direct labor requirements in the airframe manufacturing industry. These findings, to be discussed in the literature review, have resulted from

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[2] A cumulative production and production rate model is a learning curve model which includes a second independent variable, namely production rate.

investigations like those by Colasuonno, Johnson, Orsini, Smith, and Congleton and Kinton.

Like the basic learning curve model, a cumulative production and production rate model should apply to estimating direct labor requirements in industries other than airframe manufacturing. Herein lies the problem for research.

Research Problem Statement

The applicability of a cumulative production and production rate model for estimating direct labor requirements for nonairframe products, such as avionics, is not known.

Research Objectives

The objectives of this research are: (1) to determine if there is an effect by production rate changes on production direct labor requirements for avionics production, and (2) to evaluate the predictive ability of a cumulative production and production rate model for eighteen months into the future. These objectives are specified by the Air Force Plan, COPPER IMPACT II, and sponsored by the Air Force Business Research Management Center at Wright-Patterson Air Force Base, Ohio. In meeting these objectives, additional information relevant to the scope and applicability of the model will be revealed.

Overview

With the problem narrowed and the objectives outlined, the next chapter is devoted to a review of past research approaches and findings. Chapter III will discuss the research hypotheses and the methodology for testing these hypotheses. A brief summary of assumptions and limitations about methodology will close chapter III. Chapter IV will discuss data analysis and evaluation. Finally, Chapter V will contain the summary, conclusions, and recommendations of this research.

CHAPTER II

A HISTORY OF LABOR REQUIREMENT ESTIMATING MODELS

This chapter traces a necessarily abbreviated history and development of traditional learning curve models over the past fifty years and more. Of particular interest, of course, are the recent models investigated by Johnson, Orsini, Smith, and Congleton and Kinton. The results of those investigations, to be discussed momentarily, were compatible. That compatibility justifies this research. Smith's investigations receive particular attention since they form the basis for this replication of his model and its extension to avionics production.

The Early Years

This section outlines a chronological history of learning curve[3] models. While not exhaustive, the outline is intended to provide one view of why learning curves have flourished.

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[3]The term "learning curve" is used in this research rather than any of the alternative terms because the term "learning curve" is common in the United States Air Force and the preponderance of the literature reviewed. More importantly, however, "learning curve" is much more restricted than are broader concepts, such as "improvement curve" and "experience curve," as discussed by Cochran (6:63-66) and Cheney (5:1-3), respectively.

T. P. Wright of the Curtiss-Wright Corporation is generally regarded as the pioneer of learning curves (6:49). He began his research as early as 1922 (5:48). Shortly thereafter, reports of the use of learning curves were recorded. By 1925 learning curve tables were being used at McCook Field, Dayton,Ohio (6:50). In 1930 Boeing used log-log cost trends during its negotiations with the Bureau of Aeronautics (6:50).

In February 1936, T. P. Wright's article, "Factors Affecting the Cost of Airplanes," was published in the Journal of Aeronautical Sciences (9:9). In that article Wright set forth the now well-known cumulative average cost curve. Consequently, the potential of the learning curve as a modeling device was enhanced. This enhancement resulted despite, or perhaps because of, the conditions under which some of the data were collected. For example, all of the 1930 Boeing data came from projects that were similar. Each project was conducted under a constant work force concept[4] and included almost identical delivery

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[4]As reported by Fisher (9:29), constant work force was defined in January 1975 by the Vice Commander of Aeronautical Systems Division, Air Force Systems Command, as follows:

"Constant work force is defined as a concept for the development of a production schedule which reflects the application of normal learning expectancy in conjunction with efficient use of manpower and funding resources. Its objective is to relate manpower buildup to productivity improvement through learning and to optimize a peak production rate which provides for

rates. In sum, all of the assumptions of the learning curve, listed in Chapter I, were apparently met.

Since 1936, the theory and application of learning curves have been treated extensively by the several leading contributors. Some of the contributors attained positions within manufacturing which allowed them to collect data resulting from changes such as accelerated delivery schedules (6:68). For example, J. R. Crawford was an important contributor to both theory and application during World War II.

Crawford's two major contributions were: (1) establishing the unit cost curve, as opposed to Wright's cumulative average cost curve, and (2) developing, or at least recording, production scheduling techniques to take advantage of learning curve cost patterns (6:49). Due to efforts by Crawford and others, the ability of the learning curve models to account for improvements in manufacturing was reaffirmed. Those improvements were consistent enough to allow prediction[5] by simple models (12:125).

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stability in manpower and funding resources application over a reasonable time period consistent with operational force and production base requirements."

[5]Fisher (9:11-12) had a thought-provoking alternative way of stating this predictability, namely that once the standard is set (e.g., an 80 percent learning curve), management will gravitate toward it through self-fulfilling prophecy.

Learning Curve Models

There are, of course, at least two variations of the learning curve model, the unit curve and the cumulative average curve. The unit learning curve model, or Beoing theory (24:2D28) can be expressed as:

$$Y = KX^b$$

where:

- Y represents the direct labor hours for the xth particular unit.
- K represents the number of manhours to build the first unit;
- X represents the cumulative number of units produced; and
- b is the exponent of the curve.

The above expression can be transformed from exponential to linear form by extracting the logarithm of each term. This straightforward transformation yields the following:

$$\text{Log } Y = \text{Log } K + b \text{ Log } X$$

In addition to now being in linear form, the simple model becomes even more appealing when one observes that as the total number of produced units doubles, the cost per unit decreases by some constant percentage (24:2D28).

The unit learning curve model, just presented, differs from the cumulative average model advanced by T.P. Wright in 1936. The only difference, however, is that the Y (or $\bar{Y}$, depending on notation) represents the cumulative average direct labor hours (6:49). This cumulative average model is also known in DOD circles as the Northrop theory (24:2D29).

Both the unit learning curve model and the cumulative average learning curve model are addressed in this research. Therefore, at least two observations are in order. First, learning curve models are basically trend concepts, but they are not time series trends.[6]

The second observation is that, although the cumulative average curve was investigated in this research, it has severe shortcomings as a general rule when used to project direct labor costs. (One needs to keep cost improvement in the aggregate distinct from the individual item, direct labor costs.) These shortcomings are due to the continual averaging of the cost variable. Cochran (6:56-59) stated the shortcomings succinctly as follows: the cumulative average cost curve masks the effect of

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[6]Time series trends are characterized mainly by their observations having some form of dependence on time (26:603-605). Some writers believe the learning curve model is a time series. For example, Johnson (13:38) cited cumulative units as a factor which changes systematically with passage of time, i.e., cumulative units are proxies for time, according to Johnson.

changes in cost, tends to overshoot when used for projecting costs, and is slow to respond to changes in costs.

Limitations of the Learning Curve Model

Probably due to its simplicity, intuitive appeal, and long history, the learning curve model is still widely used. However, the learning curve model does not take into account the exogenous changes in the rate of production. Those exogenous changes are a concern of this research, as is their effect upon the total direct labor requirements.

Concern about exogenous changes in production rate is justified by the following factors: (1) workers will adjust according to pressure to speed up or slow down production; (2) as more workers are employed, the distribution of tasks to each individual worker should narrow; and (3) at higher production rates, tooling costs and set-up costs can be more widely allocated to larger numbers of units (21:44).

Fiscal prudence dictates that each echelon within DOD strive for accurate cost prediction in order to budget, manage, and control. It naturally follows that the importance of production rates in cost estimating must be investigated fully, and that DOD buyers must consider the effects of production rate changes throughout the acquisition process (14:1).

History of Efforts to Add Production Rate Variable

The crux of this research effort rests upon the importance of the production rate as an independent variable. The addition of this independent variable to the learning curve model has been the focus of several investigations attempting to explain the effect of production rate changes in airframe production. Since this same basic idea is to be tested in avionics manufacturing, the same type model is worthy of review. Therefore, the development of cost-estimating models and the findings of several investigators are reviewed. The majority of these individuals were concerned with production rates in airframe production. Various findings which tend to support the importance of the production rate variable will be discussed, with obvious emphasis on Smith's generalized model. Some dissenting views about the importance of the production rate will then be discussed. Finally, a summary of the literature review will close this chapter.

Colasuonno. Colasuonno (7:66-74) discussed seven airframe manufacturers whose use of the learning curve model entailed a cumulative production and production rate model (at least subjectively) in at least five of the seven cases. Colasuonno pointed out:

> If accurate quantification is possible, standardization of the definitions and categorization of factors involved, as well as a systematic method of

application, should produce better, if not highly accurate estimates [7:75].

Thus, even as early as 1967 systematic addition of the production rate variable to learning curve models was being considered.

Johnson. To predict labor requirements for rocket motors, Johnson used an additive model as follows:

$$Y = A+BX_1+CX_2^{-Z}$$

where:

- Y represents the direct labor hours per month
- X_1 represents the production rate in equivalent units per month
- X_2 represents the cumulative units produced as of the end of each month, and
- A, B, and C coefficients determined by regression; Z is assigned various values until an optimum regression coefficient of determination or R^2 is found (13:30-32).

Johnson regressed direct labor hours by month as a linear function of production rate in equivalent units per month and as a power function of cumulative units of production as of the end of the month (13:30). He expected the Z value of the power function to approach the exponent for the standard learning curve model. Instead it was

equivalent to a 20 percent "slope,"[7] versus the typical 80 percent "slope" (13:37). Johnson concluded that this lower equivalent "slope" resulted from interaction among extraneous variables (13:38). Consequently, Johnson reported that the production rate was important in determining the direct labor requirements in three of four sets of data. He excluded the third data set and reasoned that its low R^2 was probably attributable to deficient methods in labor hour accounting procedures. Of the remaining data sets, Johnson obtained two good results and one fair result (13:34).

Orsini. Orsini (20:54-80) tested Johnson's rocket motor model by using airframe data from the C-141A program. Orsini's procedure employed regression of Johnson's model both with and without the independent variable of production rate. After those regressions Orsini (20:66) transformed Johnson's additive model into a multiplicative one as follows:

$$Y = e^{B_0} \cdot X_1^{B_1} \cdot X_2^{B_2}$$

[7]Because the "slope" of a learning curve is a mathematical misnomer, it cannot be related to the definition of slope in a straight line. The "slope" of a learning curve is equal to 100 minus the rate of learning, where the rate of learning is the constant percentage decrease in cost per unit as the total quantity of units produced doubles (19:53).

Y represents the direct labor hours per quarter

X_1 represents the number of units produced per quarter

X_2 represents the cumulative number of units produced as of the end of each quarter

B_0, B_1, and B_2 represent regression coefficients; and e, is the base of natural logarithms.

Orsini drew three major conclusions from his studies. First, the production rate variable contributed importantly to the explanatory power of both the additive and multiplicative models. Second, the multiplicative model was a better predictor than the additive model because the estimate of the Z value was eliminated. Third, Orsini indicated that inclusion of the production rate in the learning curve model yielded more accurate results and would possibly lead to significant revisions and improvement of cost estimating (20:83-86).

Smith. Noting that a production rate variable was not included in any generally accepted model for estimating costs for airframe production, Smith developed a model to include this variable. He designed a model whose variables could be tailored within a given airframe production program and whose coefficients could be adjusted to accommodate a continually changing data base during production. The intent of Smith's efforts was to provide

for more accurate prediction of direct labor requirements for additional airframes within a given program (21:3,56-57).

For a model, Smith selected a modified version of Orsini's multiplicative model, specifically as follows:

$$Y_i = B_0 \cdot X_{1i}^{B_1} \cdot X_{2i}^{B_2} \cdot 10^{e_i}$$

where:

Y_i represents the unit average direct labor hours needed to output each pound of airframe in lot i.

X_{1i} represents the cumulative learning accrued from experience on all airframes of the same type through lot i.

X_{2i} represents the production rate of lot i for all airframes of the same type.

e_i represents the variation of each dependent variable which is not explained by the two independent variables.

B_0, B_1, B_2, are parameters in the model (21:43).

Smith chose to include the production rate in the multiplicative model because other investigators suggested that its inclusion could enhance predictive ability of a model. Moreover, Smith reported that its inclusion facilitated

multiple regression, and he indicated that he had achieved good results from the model by using test data (21:43).

From there, Smith linearized the multiplicative model and used proxies for the production rate variable. The linearized model thus became:

$$\text{Log } Y_i = \text{Log } B_0 + B_1 \text{ Log } X_{1i} + B_2 \text{ Log } X_{2i} + e_i$$

In this form, linear in each term, it became possible to use multiple linear regression (8:19).

Smith calculated two proxies for the production rate variable, namely "lot average manufacturing rate" and "lot delivery rate." He defined the first as the number of airframes in a lot divided by the lot time span, where lot time span was the time between release date from the lot for the first airframe and the completion date for the last airframe in the lot. The second proxy, the "lot delivery rate," was the actual monthly airframe acceptance rate (21:11-13).

Smith employed a second, or "reduced," model which was merely his first, or "full" model, minus the delivery rate variable. In other words, his "reduced" model was a unit learning curve model, as follows:

$$Y_i = B_0 \cdot X_{1i}^{B_1} \cdot 10^{e_i} \qquad (21:29)$$

Hence, once both models were linearized, regression of historical data with each model allowed Smith to identify

the contribution of predictive ability by the independent variable of production rate (8:19).

Smith evaluated data from the F-4, F-102, and KC-135A airframe production programs. After extensive investigation (and statistical tests in a majority of the sixteen test situations), Smith concluded the following: (1) in each case, the production rate variable was negatively correlated with unit direct labor requirements; (2) both proxies which he calculated were important contributors to the full model's predictive ability; (3) as evidenced by the R^2 values he obtained, the full model more closely fit the data than did the reduced model (21:142-146). Tables 1 and 2 summarize Smith's regression analysis and predictive ability test results.

Congleton and Kinton. Congleton and Kinton replicated Smith's study using data from T-38 and F-5 production programs. Their methodology was the same as Smith's, and their conclusions largely reaffirmed his findings.

Congleton and Kinton validated the predictive ability of Smith's model, and their research was highlighted by the following conclusions: (1) there is a negative correlation between production rate and direct labor requirements, as evidence by negative B_2 coefficients in each test situation; (2) the proxies were important contributors to the predictive ability of the full model; and (3) the full model more closely fit the data than did

TABLE 1

SUMMARY OF SMITH'S REGRESSION ANALYSIS[a]

Test Situation No.	Data Points	R_f^2 (actual)	R_r^2 (actual)	B_0	B_1	B_2
1	57	0.978	0.928	masked[b]	-0.261	-0.169
2	55	0.973	0.904	"	-0.246	-0.183
3	55	0.966	0.904	"	-0.257	-0.161
4	42	0.853	0.585	"	-0.230	-0.157
5	42	0.820	0.585	"	-0.229	-0.136
6	42	0.889	0.618	6.328	-0.221	-0.148
7	42	0.851	0.618	7.601	-0.219	-0.127
8	42	0.744	0.658	9.016	-0.279	-0.112
9	42	0.733	0.658	10.400	-0.278	-0.097
10	50	0.979	0.961	38.371	-0.299	-0.158
11	42	0.979	0.959	47.290	-0.344	-0.144
12	96	0.958	0.971	13.133	-0.453	-0.164
13[c]	7	0.974	0.903	0.674	-0.165	-0.305
14[c]	7	0.971	0.903	1.123	-0.233	-0.222
15[c]	7	0.994	0.964	13.338	-0.608	0.361
16[c]	7	0.992	0.964	7.303	-0.527	0.262

[a]The total production hours per pound were considered proprietary by the manufacturer, and these coefficients were masked in the published version of Smith's research (21:65).

[b]Smith's methodology, production rate proxies, and R^2 versus R^2 (actual) are all recapped in Chapters II and III of this research. The subscripts for R^2 are as follows: f stands for full model; r for the reduced.

[c]Impractical for test situations.

TABLE 2

SUMMARY OF SMITH'S PREDICTIVE ABILITY TEST RESULTS

Test Situation No.	Percentage Deviation[a]	
	Full Model	Reduced Model
1	-2.6	14.5
2	2.2	13.6
3	Not Reported	13.6
4	1.8	5.3
5	3.1	5.3
6	-7.8	Not Reported
7	"b"	Not Reported
8	-0.7	1.1
9	-4.2	1.1
10	-1.1	5.6
11	3.5	Not Reported
12	2.2	-3.3
13-16	"c"	"c"

[a]These tests were conducted as described in Chapter IV of this research (21:56). All percentages are rounded to nearest tenth.

[b]Smith reported the results were deviations greater than those for test situation 6, but he did not report a value (21:96).

[c]Smith reported that predictive ability tests were impractical for situations 13 through 16 because observations were limited to seven (21:131).

SOURCE: (21:71-131).

the reduced model. In one of the thirty test situations they reported that R^2 (actual) was higher for the reduced model than for the full model, but by less than 1 percent (8:91-93).

Other Opinions and Findings

Rarely is any series of findings unanimous, and the effect of production rate changes is no exception. In addition to the test situation just mentioned, wherein Congleton and Kinton reported they arrived at the higher actual R^2 for the reduced model, other investigators do not agree with the hypothesis that production rate changes are important. These dissenters include at least four investigators.

Asher. Asher (1:86-87) evaluated empirical data for various airframe production programs. Rather than attempt to statistically segregate the effect of production rate and the effect of cumulative production on unit costs, he subjectively evaluated the effect of the production rate on direct labor hours. Asher noted there were two ways that the rate of production could influence unit labor cost. It could affect the machine set-up time, that is the number of hours charged to each unit of production. And it could, according to Asher, affect the number of subassemblies in a manufacturing process which, in turn, could lead to an effect on the number of hours of

subassembly work charged to a unit. Asher concluded that production rate was not very important when compared to the effect of cumulative production. He also concluded that aside from set-up time and subassemblies, there was little reason to expect a significant difference in unit hours per month, whether 200 or 30 units per month was the manufacturing rate.

Large, Hoffmayer, and Kontrovich. These three investigators evaluated cost for major defense buys of airframes at varying production rates during an effort to develop a general cost model which, according to Smith (12:30), was of the form:

$$Y_i = A \cdot w^B \cdot s^C \cdot r^D$$

where:

Y_i represented the cumulative direct manufacturing labor hours through unit i

w represented the program average DCPR weight[8]

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[8]It is the practice of the government to furnish many components to a contractor building a new aircraft. Engines, electronic systems, wheels, brakes, tires, and batteries exemplify these types of components. To have a common basis for comparing weight based on cost estimating relationships, government and industry have agreed to a Defense Contractor Planning Report (DCPR) weight that excludes the government-furnished equipment, fuels and lubricants. The DCPR weight was formerly termed the Airframe Manufacturer Planning Report (AMPR) weight (21:11-12).

s represented the maximum design airspeed in knots

r represented the production rate expressed as the acceptance span in months for the first i airframes (NOTE: For their investigation, Large, Hoffmayer, and Kontrovich chose i at an arbitrary 100 or 200).

A, B, C, and D were model parameters.

Large, Hoffmayer and Kontrovich concluded that the effect of production rate change could not be confidently predicted based on their findings and analysis (14:50-51). Smith (21:30-31) commented that their use of an acceptance span for production rate almost of necessity obscured the effect of production rate change.

Large, Hoffmayer, and Kontrovich also investigated the Johnson rocket motor model. Their results were different than Orsini's. They concluded that:

> In none of the programs did the inclusion of production rate improve the coefficient of determination, R^2, by as much as one percent over what was obtained using cumulative quantity alone [14:49].

Hirsch. Orsini (20:53) reported that Werner Z. Hirsch did not include production rate in his study of empirical data in machine-tool manufacturing because he believed changes in production rate were already accounted for in cumulative output.

Summary

Almost all of the literature reviewed contained an interest in the relationships between production rate and direct labor requirements. The controversy which clearly shows up is whether or not there is a significant relationship between production rate and direct labor requirements. A dominant theme of the literature, dissensions noted, was that production rate is an important contributor to the predictive ability of a learning curve model. This research, therefore, is aimed at two particular areas:

1. Is there a significant relationship in avionics production between production rate and direct labor requirements, and

2. Is production rate in avionics production an important contributor to the predictive ability of a learning curve model?

The next chapter outlines the methodology to be used to answer these questions.

CHAPTER III

RESEARCH METHODOLOGY

This chapter outlines the research hypotheses and the methodology used to test them. The chapter is divided into six sections as follows:

1. Objectives and approach;
2. The variables;
3. Model definitions and assumptions;
4. Research hypotheses;
5. Data collection and treatment; and
6. Summary.

Objectives and Approach

The objectives of this research were: (1) to determine if there was an effect by production rate changes on production direct labor requirements for avionics production, and (2) to evaluate the predictive ability of the cumulative production and production rate model for eighteen months into the future. In meeting these objectives, additional information relevant to the scope and applicability of the cumulative production and production rate model was revealed.

The approach was to collect historical production data from avionics manufacturing and evaluate those data via replication of Smith's model, previously described

in Chapter II. As in Smith's research, as well as in Congleton and Kinton's, the objective was not to formulate a generalized cost model. Rather, the intent of the research was to evaluate the avionics production data as a means of adjusting an appropriate form of the cumulative production and production rate model to specified programs outside airframe manufacturing. Thus, the value of a properly adjusted model, when compared with the learning curve model, lies in its potential ability to predict more accurately the direct labor requirements in avionics production.

The Variables

Three avionics production variables and their relationships were evaluated. The variables were continuous and identified as follows:

1. The direct labor hours variable.
2. The cumulative output variable.
3. The production rate variable.

Although any one of the variables could be considered dependent on the other two, direct labor hours was designated the dependent variable.

The Direct Labor Hours Variable

Expressed as direct labor hours, the dependent variable comes primarily from assembly, fabrication, and

testing. It also included hours devoted to preventive maintenance, inspection, quality assurance, material handling, repairing, troubleshooting, and packing. All of the hours of the dependent variable pertained to in-house operations.

The Cumulative Output Variable

Cumulative output in this research was the cumulative number of equivalent units of production as of the end of an accounting month.

The Production Rate Variable

The production rate variable in its basic form was the number of equivalent units[9] produced during an accounting month.

Model Definition and Assumptions

Two types of models were investigated during analysis of each model, one through five. The first type of model, the learning curve model, hereafter is referred to as the reduced model. The second type of model, the cumulative production and production rate model, hereafter is referred to as the full model. Both types of models were

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[9]An equivalent unit is a common denominator which represents the total work of a department or process in terms of full completed units. As the ASPM No. 1 (24:2D31) indicates, for continuous process production the costs are usually attributed to equivalent, not actual units. Related discussions are found in Cochran (6:435-437) and Johnson (13:34).

investigated by Smith (21:42-43). Both the reduced and full models were evaluated by Congleton and Kinton (8:31-32). Finally, a discussion of each type of model is found in Neter and Wasserman (17:256-258).

Model Definition

The reduced models are forms of the standard learning curve where:

$$Y_i = B_0 \cdot X_1^{B_1} \cdot 10^{e_1}$$

The full models include a second independent variable for the production rate where:

$$Y_i = B_0 \cdot X_1^{B_1} \cdot X_2^{B_2} \cdot 10^{e_1}$$

Common terms in each type of model have the same meaning and are described as follows:

Y_i represents the direct labor hours variable;

X_1 represents the cumulative output variable;

X_2 represents the production rate variable; and

e_i represents the variation in each dependent variable value that is not explained by the two independent variables; finally

B_0, B_1, and B_2 are model parameters.

The notation and precise definitions of each model and its variables will be described and summarized in tabular form near the end of this chapter.

Assumptions of the Models

The first assumption of the models concerns the potential loss of precision in the data, to be explained shortly. To facilitate multiple linear regression, the method used to investigate each model, terms of the models were linearized (17:123-128) by taking the common logarithm of each term.

In log-linear form then, the reduced models become:

$$\text{Log } Y_i = \text{Log } B_0 + B_1 \text{ Log } X_1 + e_i$$

and the full models become:

$$\text{Log } Y_i = \text{Log } B_0 + B_1 \text{ Log } X_1 + B_2 \text{ Log } X_2 + e_i$$

It is pointed out that such a transformation to logarithmic form does affect the least squares estimators. According to Neter and Wasserman (17:130):

> When transformed models are employed, the estimators b_0 and b_1 obtained by least squares, have the least squares properties with respect to the transformed observations, not the original ones.

The assumption was made, therefore, that transformation did not introduce a significant loss in the precision of the data.

A second assumption, or set of assumptions, related to the error terms. To allow for statistical significance testing of the regression results, the error terms (e_i) in the logarithmic domain were assumed to be normally distributed with a mean of zero and a constant variance.

Moreover, the error terms were assumed to be independent of each other and of the independent variables (17:30-31).

It is emphasized that as a practical matter, one should not be unduly concerned with normality of the error terms. That is not to be construed as a rationalization for a model not fitting a data set; rather, in this research the prime objective of the model, its predictive ability, was borne in mind as the residuals were subjectively evaluated. In their statistical text, Neter and Wasserman (17:48) stated that:

> . . . unless the departures from normality are serious, particularly with respect to skewness, the actual confidence coefficients and risks of error will be close to the levels for exact normality.

The third assumption of the model, to be discussed momentarily, stemmed from a complication of regression analysis. The complication which did occur in this research was multicollinearity[10] between the two independent variables. The likelihood for this was evidenced by viewing the learning curve theory in reverse. That is to say, if labor hours are held constant while cumulative output increases, the rate of production goes up because each successive item requires less production time (8:33). Moreover, the likelihood of multicollinearity was evidenced initially by the very nature of the data (17:255).

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[10]Multicollinearity is treated separately in Appendix B.

In regression analysis, high multicollinearity between or among independent variables may cause the individual regression coefficients to vary widely among samples. A possible consequence is rejection of the significance of a coefficient when, in fact, the coefficient is significant (17:341). Despite this possible consequence, the predictive ability of the model in the short range may not be importantly impaired. Because the model's biggest asset was its predictive ability in this research, the model was still of value, as evidenced by both statistical and criterion tests. The production rate as an added explainer of variation in the dependent variable was subjectively evaluated by comparing the predicted values of direct labor hours made by both the full and reduced models with observed values (8:33). In sum, the third assumption of the models was that the varying degrees of multicollinearity did not importantly impair their short-range predictive abilities.

Research Hypotheses

This research tested two hypotheses. The first hypothesis was that the production rate was an important explainer of variation in direct labor requirements for avionics production when included in an appropriate model. The second hypothesis was that for eighteen months into the future the predictive ability of the full model was better than the predictive ability of the reduced model.

Research Hypothesis One

The first research hypothesis was tested in two steps. The first step was examination for statistical significance of the model's regression coefficients determined by regression analysis of historical avionics production data. The second step was use of two criterion tests to evaluate the appropriateness of the model for the data. The model tested was the full model in log-linear form. During those tests, the dependent variable was subjected to regression analysis. The independent variables in turn for X_1 and X_2, were the common logarithms of the cumulative output and production rate variables.

Statistical Hypothesis One (A)

Statistical Hypothesis One (A) (8:35) stated that the cumulative output variable and the production rate variable were related to labor hours as indicatd in the model. Putting the null hypothesis and its alternate in statistical form yielded the following:

$$H_0: \ B_1 \text{ and } B_2 = 0$$

$$H_a: \ B_1 \neq 0 \text{ and/or } B_2 \neq 0$$

The decision rule was as follows: The null hypothesis was rejected if the test statistic F_{ratio} was

greater than the critical statistic[11] F_c at the 0.05 level of significance.

For this statistical test, F Ratio = MSR/MSE [MSR = SSR/(p-1); MSE = SSE/(n-p)] where, in logarithmic form:

MSR represents the mean of the regression sum of squares;

MSE represents the mean of the error (or residual) sum of squares;

SSR represents the regression sum of squares;

SSE represents the error (or residual) sum of squares; and

n represents the number of observations.

p represents the number of parameters in the model

In sum, the F Ratio compares the explained variance (MSR) with the unexplained variance (MSE), and it indicates the relationship between direct labor hours and the set of two independent variables, cumulative output and production rate (17:45,79,227-228).

Statistical Hypothesis One (B)

Statistical Hypothesis One (B) (8:36) was designed to test ability of the production rate variable, when combined with the cumulative output variable, to explain

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[11]F_c values were extracted from Neter and Wasserman's F-distribution tables (17:807-813).

additional variation in the direct labor hours. Statistically speaking, the B_2 coefficient was hypothesized to be nonzero at the 0.05 level of significance. In null and alternate forms, statistical hypthesis one (B) was as follows:

$$H_0: \; B_2 = 0$$
$$H_a: \; B_2 \neq 0$$

As earlier, the null hypothesis was rejected if the test statistic F^* was greater than the critical statistic F_c at the 0.05 level of significance.

For this test, the F^* was calculated by determining the increase in explained variation of the dependent variable that was attributed to the introduction of the logarithm of the production rate variable into the reduced model. In specific terms:

$$F^* = \frac{R}{(1-R^2)/(n-k-1)}$$

where:

R^2 represents the change in the explained variation;

n represents the number of observations;

k represents the total number of regressors; and

$n-k-1$ represents the degrees of freedom in the unexplained variation (26:435)

Because the two independent variables in this research were correlated, an observation is in order. It is noted that, while the increase in explained variation of the dependent variable can be attributed to the introduction of the logarithm of the production rate variable, not all of that increased explained variation is uniquely attributable to the additional variable. Rather, the incremental change in explained variation of the dependent variable is that variation associated with the introduction of the logarithm of the production rate variable, given that the logarithm of the cumulative output variable is already included in the model (17:253).

The crux of the dilemma concerning partitioning incremental changes in variation when the independent variables are correlated was described by Neter and Wasserman (17:253) as follows:

> When independent variables are correlated, there is no unique sum of squares which can be ascribed to an independent variable as reflecting its effect in reducing the total variation in Y.

Criterion Test One (A)

Whereas Smith (21:50-52) used a subhypothesis to evaluate the appropriateness of the model, Congleton and Kinton (8:37-39) used a criterion test. Both methods yield the same objective in determining the appropriateness of the model. Because determination of the appropriateness of the model was necessarily partially

subjective, two criterion tests were used in this research. The model was not rejected as inappropriate if the first criterion test, to be described momentarily, revealed an inability to reject assumptions about the normal distribution of the residuals, their independence, and their constant variance (17:240).

Whether the residuals are normally distributed may be checked in at least two ways. They may be plotted on normal probability paper to determine if they deviate substantially from a straight line. Or the residuals can be classified into a histogram (17:107). Because the plot on normal probability paper was faster, that method was used in this research. When the deviations from a straight line were not substantial, that is, when the departures were not serious, as noted by Neter and Wasserman, the assumption of normality was not rejected.

Neter and Wasserman (17:105,240) discussed independence of residuals in the following way. To determine if the residuals are independent of each other and of the independent variable, they can be plotted against each independent variable to test for cyclic recurrences or trends. If there are no such recurrences or trends and if the residuals fluctuate randomly above and below the line formed by the plot of the predicted values, the assumptions of independence will be considered met. In this research, that method of checking independence was used.

The absence of recurrences, trends, and patterns was the criterion used for ability or inability to reject the assumption of independence of residuals.

Acceptability of the assumption of constant variance of residuals was evaluated by plotting the residuals against the value of the dependent variables predicted by regression. If the plot revealed an even distribution and if a heavy majority of the residuals were within one standard error of the estimate, the assumption of constant residual variance was considered met (17:102-103).

In sum, the evaluation of the residuals was important when the appropriateness of the model was being evaluated. Such an evaluation, in fact, was a prerequisite for determining the model to be either appropriate or inappropriate for a data base (17:43). The appropriateness of the model could not be rejected if an analysis of the residuals reflected the properties of constant variance, independence, and normality assumed for the theoretical residuals.

Criterion Test One (B)

A second criterion test (21:52-54) used to evaluate the appropriateness of the model was the test of the multiple coefficient of determination, commonly termed R^2 (17:228). The R^2 measures the proportion of variation in

the dependent variable explained by the regression model. Put another way, R^2 is equal to one minus the quotient of SSE/SSTO (17:228). The error sum of squares, SSE, was just defined under statistical hypothesis one (A). More specifically, SSE is the summation of all squared residuals (17:77). The SSTO is the total sum of squares and is equal to the summation of the squared differences between each observed value and the mean of the dependent variable (17:77).

The distinction of R^2 in the models of this research was that, due to transformation of the rational values to logarithms prior to regression analysis, the R^2 was really the proportion of explained variation to total variation in the logarithms of labor hours, not labor hours per se. To make R^2 more meaningful in terms of actual labor hours, Smith (17:53) developed a second statistic for use in his research. This second statistic, termed R^2 (actual) was computed by a procedure analogous to the one just described. The only difference was that for the R^2 (actual) the SSE and SSTO computations were in terms of observed labor hours.

The calculations of variation in observed labor hours was made possible because the predictions in logarithms were transformed to labor hours in rational numbers, the form of the original variable. These transformations were included, of course, as integral parts of

the computer program located and described in Appendix A. And as earlier stated, it was assumed that such transformations did not involve a significant loss in the precision of the data.

Because R^2 (actual) measured the proportion of variation in labor hours explained by the regression model, a high R^2 (actual) was a practical consideration in determining which models were in fact appropriate. Accordingly, the criterion test chosen for R^2 (actual) in this research was as follows. If the regression results of the model included an R^2 and an R^2 (actual) exceeding 85 percent, the model could not be rejected as inappropriate. If the model could not be rejected, the predictive ability was then tested under Research Hypothesis Two.

Research Hypothesis Two

The second research hypothesis was that for eighteen months into the future the predictive ability of the full model was better than the predictive ability of the reduced model. As in research hypothesis one, this second research hypothesis was evaluated using both a statistical hypothesis to test statistical significance, and a criterion test for practical significance.

The statistical test used in this portion of the research was a simple Z test of the differences between two means. To conduct this test, future predictive

ability was simulated using a stepwise truncation of historical data. Smith (21:56) described the process as follows:

> In a real application of the model, the prediction would be beyond the range of the historical data. The only way to test the accuracy of the prediction would be to wait and see how many hours it takes to build the next airframe lot. To simulate this situation, the regression coefficients in the model are estimated with the last few observed data points omitted. Then using the new model, omitted values (which are known but not used in estimating the model coefficients) are predicted. Comparisons are then drawn between the actual and predicted hours as a subjective measure of predictive ability.

Statistical Hypothesis Two

To perform a test for statistical significance it was necessary to formalize the subjective comparisons of actual and predicted direct labor hours into an objective measure of difference. This was done by taking the absolute value of the difference between the actual and predicted direct labor hours occurring with the full and reduced models in each test situation. These absolute deviations were then separately summed for each model in all test situations. The mean and variance of the two distributions of absolute deviations were then calculated. A statistical comparison of the two distributions was then possible to test the hypothesis that the predictive ability of the full model was better than the predictive ability of the reduced model. This hypothesis could be stated more precisely as: The average absolute deviation

of the full model ($|\bar{D}_F|$) is significantly less than the average absolute deviation of the reduced model ($|\bar{D}_R|$). Thus, in null and alternate forms, statistical hypothesis two becomes:

$$H_o: \quad |\bar{D}_R| \leq |\bar{D}_F|$$

$$H_a: \quad |\bar{D}_R| > |\bar{D}_F|$$

The decision rule using the Z statistic (17:12-17) becomes:

$$\text{Reject} \quad H_o \text{ if } Z > Z_c(.05)$$

where

$$Z = |\bar{D}_R| - |\bar{D}_F| / \sqrt{(S_R^{\,2}/N) + (S_F^{\,2}/N)}$$

and

$S_R^{\,2}$ = variance of the distribution of deviations obtained with the reduced model;

$S_F^{\,2}$ = variance of the distribution of deviations obtained with the full model;

N = the number of test situations; and

Z_c = critical Z value obtained from a table of critical values for the standard normal distribution.[12]

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[12]Z_c values for the one-tailed test were extracted from Neter and Wasserman's standard normal distribution

The Z statistic used here refers to the use of the standard normal distribution. The Student's t distribution should be used in situations where the number of test situations was small (less than 60). In this research, however, the number of test situations was anticipated to be sufficiently large to permit the use of the standard normal statistic. In applying either a Z or t test, the assumptions were made that the deviations were normally distributed and random. Since these assumptions were also made in computation of the regression statistics in research hypothesis one, the authors believe the assumptions were compatible.

In evaluating the predictive ability of the full and reduced models, statistical significance alone does not indicate the value of the model. For example, if the full model overestimated the direct labor requirements on the average by 100 percent and the reduced model overestimated the same direct labor requirements on the average by 150 percent, this difference in the two means could easily be proved to be statistically significant. However, neither model would be of any practical value in estimating costs. Accordingly, a criterion test was developed

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table, Table A-1 (17:803). A 5 percent level of significance for the Z_c was chosen because of the prevalent use of the 95 percent confidence interval in the statistical and research literature reviewed. In practice, of course, the user would want to choose a level of significance consistent with the risk he would be willing to take.

to address this practical aspect of the importance in the predictive ability of the full and reduced models.

Criterion Test Two

To perform the criterion test, the individual deviations computed for the full and reduced models in each test situation under statistical hypothesis two were converted into a measure of deviation expressed as a percentage of the actual direct labor hours. The authors believed that the use of percentages in this test would facilitate comparison of results between programs whose values for direct labor hours were relatively small (e.g., hundreds of hours) and programs whose values for direct labor hours were relatively large (e.g., thousands of hours). Two arbitrary categories were then established for the deviations.

These categories provided a basis for comparison of the predictive ability of the two models. When the percentage deviations were limited to 10 percent or less, the predictive ability was categorized as good; and when the percentage deviations were limited to 5 percent or less, the predictive ability was categorized as excellent. The number of test situations in which the percentage deviations fell into these categories was then separately summed for the full and reduced models. Totals for each category and model were then subjectively compared and the model with the greater total number of good and

excellent predictions was judged to have the better practical predictive ability.

This criterion test, while very simple, permitted a comparison of how well each model could predict the future.

Data Collection and Treatment

Because the approach of this research was to replicate Smith's full model and extend it to production data outside airframe manufacturing, accessiblity was the primary factor in selecting data. The data were historical and represented two individual avionics production programs. Both programs were already established and ongoing. One data set contained thirty-nine data points; the other, forty-seven. All data were provided directly by the manufacturer under the following labels: (1) number of workdays per accounting month; (2) equivalent units of production per accounting month; and (3) average direct labor hours per equivalent unit per accounting month. Treatment of the data for the dependent variable and for the independent variables was necessary to tailor the data to the models.

Treatment for Labor Hours

Data for labor hours were provided as the average direct labor hours per equivalent unit per accounting month. The following breakout was used in this research.

1. Y_u stands for the unit cost dependent variable. Where Y_u is used, the dependent variable represents the average direct labor hours per equivalent unit per accounting month, as provided by the manufacturer.

2. Y_c stands for the cumulative average cost dependent variable. Where Y_c is used, the dependent variable represents the cumulative average direct labor hours per equivalent unit. This distinction between Y_u and Y_c was noted by Smith (21:9) as a common one, although his investigations centered on the unit cost dependent variable. To derive Y_c requires two steps, given the data as provided by the manufacturer. First, the equivalent units produced during an accounting month are multiplied by the average direct labor hours per equivalent unit per accounting month. Second, the resulting product is then divided by the number of cumulative units to date.

3. Y_t stands for the total cost dependent variable. Where Y_t is used, the dependent variable represents the total direct labor hours per accounting month, as used by Johnson (13:37). Y_t is derived simply by taking the product of equivalent units per accounting month and average direct labor hours per equivalent unit per accounting month.

Treatment for Cumulative Output (Lot Plotpoints)

For all models employing the average hours per equivalent unit per accounting month variable (Y_u) as the dependent variable, the cumulative units produced, X_1, had to be adjusted to avoid incorrect conclusions about the relationship between cost and quantity (3:7-8). This necessary adjustment makes both practical and intuitive sense for at least two reasons. First, unlike a cumulative average cost curve which by definition is smoothed, the unit cost curve falls extremely fast from its initial point for unit one. Second, the whole idea behind establishing a correct plotpoint is to approximate the learning curve applicable to production in a given lot (6:112).

Accordingly, the X_1 variable, cumulative output, was adjusted in three steps where appropriate, as follows:

1. Midpoints for each of the lots (or each accounting month's number of equivalent units produced, as provided by the manufacturer) were calculated, using the first and last units of each lot, summing them and dividing the result by two to arrive at a midpoint for that lot (6:111).

2. All midpoints derived in step one were then entered into the data base in place of the original X_1 independent variables, as provided by the manufacturer.

The data were then regressed and a learning curve "slope" obtained.

3. Finally, in accordance with established learning curve technique (21:61), the midpoint for the first lot was replaced with the tablular value (3) which corresponded to the size of the first lot within the table provided for the "slope" obtained from the regression in step two.

For the second and succeeding lots, the plotpoints were simply actual midpoints (6:122). After treatment to this point, then, all plotpoints conformed conceptually to the lot cost average. This conformance was the objective, to avoid incorrect conclusions about the relationship between cost and quantity (3:7-8).

Treatment for Production Rate

As provided by the manufacturer, the data contained no production rate, other than the equivalent units produced per accounting month. Therefore, this independent variable was treated as follows:

1. X_{2a} stands for the daily average production rate variable. Where X_{2a} is used, the production rate variable represents the daily average production rate per accounting month. It is derived by dividing the equivalent units produced during an accounting month by the

number of working days in that accounting month, as provided by the manufacturer.

2. X_{2c} stands for the cumulative average production rate variable. Where X_{2c} is used, the production rate variable is the cumulative average of the daily average production rates, whose derivation was just described.

3. X_{2m} stands for the monthly production rate variable. Where X_{2m} is used, the production rate variable simply represents the number of equivalent units produced during an accounting month, as provided by the manufacturer.

With the dependent variable designated and the distinctions among variables and the forms of each variable outlined, Table 3 is provided as a quick reference. The table should prove useful, as various cost models and their variables are introduced, discussed, analyzed, and referred to throughout the remainder of this paper. Moreover, the table serves as a reminder that in searching for a predictive model, one must look at as many potential treatments of data as time and other resources allow.

Beyond the treatment of the variables, the treatment of the data base itself was of importance. In lieu of sample, the assumption of population census was used. Therefore, statistics derived for each individual population were descriptive of only that population. Those

TABLE 3

OVERVIEW OF MODELS AND THEIR VARIABLES

Model	Direct Labor Hours	Cumulative Output	Production Rate
1	Y_u	X_{1a}	X_{2a}
2	Y_c	X_1	X_{2a}
3	Y_u	X_{1a}	X_{2c}
4	Y_c	X_1	X_{2c}
5	Y_t	X_1	X_{2m}

Where:

in Y: u is unit cost, or average direct labor hours per equivalent unit per accounting month;

c is cumulative average cost, or cumulative average direct labor hours per equivalent unit; and

t is total cost, or total direct labor hours per accounting month.

in X_1: a is cumulative output adjusted, or adjusted plotpoints for cumulative production in equivalent units. The absence of an alpha subscript indicates no adjustment was required.

in X_2: a is average rate, or daily average production rate per accounting month;

c is cumulative average rate, or cumulative average of the daily average production rates per accounting months; and

m is monthly rate, or the monthly production rate, as represented by the number of equivalent units produced per accounting month.

statistics were limited by the methodology, the assumptions of the models, and the models' explanatory powers. Consequently, applicability of the regression coefficients was necessarily limited to the individual avionics program from which the coefficients were derived.

Summary of Methodology, Assumptions and Limitations

The approach was to analyze historical production data by using least squares multiple linear regression analysis. Statistical and criterion tests were structured for testing the research hypotheses.

To evaluate the first research hypothesis, statistical tests were constructed to investigate in an appropriate model the hypothesized effect of production rate changes on direct labor requirements in avionics production. Two criterion tests were used for judging the appropriateness of the model. If both statistical tests and both criterion tests were passed, the full model was validated. The conclusion to follow was that production rate was an important explainer of the variation in total direct labor requirements for avionics production.

To evaluate the second research hypothesis, the predictive abilities of the full and reduced models were compared over an eighteen month time span. A statistical test was employed to determine if the predictive ability

of the full model was significantly better than the predictive ability of the reduced model. In addition, a criterion test was used to determine which model had the better practical predictive ability. The objective here was to demonstrate that the improvement in predictive ability achieved by the full model was not only statistically significant but of practical value as well.

The strength and validity of the conclusions associated with the research hypotheses must necessarily be evaluated in terms of the assumptions and limitations inherent in the methodology. In that context, a recap of the assumptions and limitations is listed.

Assumptions

Historical data obtained from the manufacturer were recorded accurately and appropriately attributed to equivalent units they applied to.

Multicollinearity did not importantly impair the short range predictive ability of the models.

The data were accurately measured and adjusted for inequalities, such as differing sizes of accounting months, particularly for production rate and lot midpoint calculations.

Logarithmic transformation of the data to facilitate multiple linear regression analysis introduced no significant loss of data precision.

In lieu of samples, population census was assumed for each data base.

Autocorrelation did exist to some degree, but was insufficient to preclude achievement of the research objectives.

Limitations

Subjective analysis was required to evaluate the actual residuals compared to the assumptions of the theoretical residuals.

Limited number of data points (both programs fewer than sixty) resulted in reduction of statistical "leverage" (i.e., limited degrees of freedom).

The applicability of the results of the research was constrained to the particular programs whose observations constituted the data bases.

CHAPTER IV

DATA ANALYSIS AND EVALUATION

This chapter presents the unbiased results of analysis of the two avionics production programs and their data sets. It is divided into three sections. Sections one and two briefly describe the production programs, the results of hypothesis testing using the models described in Chapter III, and the major findings. The last section compares and contrasts the findings in terms of both the production program environment and the statistical results, and it evaluates the overall applicability of the cumulative production and production rate model for the two programs.

The Magnavox Data

The data obtained from the Magnavox Government and Industrial Electronics Company consisted of the production direct labor requirements for the ARC-164 radio. The data were composed of thirty-nine data points, stretching from the program's beginning in August 1975 through October 1978. This ongoing program made an ideal test situation for the extension and application of both the learning curve and the cumulative production and production rate models in the area of avionics production (8:97).

The raw data were manipulated as described in the explanation of the individual variables for each of the models presented in Chapter III. Regression analysis was performed on both the full and reduced forms of the models, and test statistics were calculated.[13] The test statistics were then compared with the critical values required and the criterion tests were applied to determine whether or not the first research hypothesis was supported. If the results obtained for a particular model supported research hypothesis one and the criterion test failed to reject the model as inappropriate, that model was then tested for support of research hypothesis two. In this manner, unnecessary testing of inappropriate models was avoided.

Analysis of Research Hypothesis One

For ease of reference and understanding, the statistical hypotheses and criterion tests for research hypothesis one are summarized and restated as follows:

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[13]The primary regression results used throughout this research were obtained through use of Smith's FORTRAN IV program (21:147-153) which was extensively modified by the authors. This modified program is listed and described in Appendix A. In addition, the program has been made available for use by government price analysts through the COPPER IMPACT Library under the file name PRODRATE.

Research Hypothesis One. Production rate is an important explainer of the variation in total direct labor requirements for avionics production when included in an appropriate model.

Statistical Hypothesis One (A). H_0: B_1 and $B_2 = 0$; H_a: $B_1 \neq 0$ and/or $B_2 \neq 0$. Reject H_0 if F Ratio is greater than F_c.

Statistical Hypothesis One (B): H_0: $B_2 = 0$; H_a: $B_2 \neq 0$; Reject H_0 if F^* is greater than F_c.

Criterion Test One (A). The model's appropriateness cannot be rejected if an analysis of the residuals indicates the assumptions of constant variance, independence, and normality are not violated.

Criterion Test One (B). The model's appropriateness cannot be rejected if the computed R^2 and R^2 (actual) exceed 85 percent.

The results of testing for research hypothesis one are presented in tabular format for each model tested. At this point it is necessary to remind the reader that all of the models were of the same basic form. That is, each reduced model contained a dependent variable representing direct labor requirements and an independent variable representing cumulative production. Likewise, each full

model contained the same dependent variable representing direct labor requirements, the same independent variable representing cumulative production, and an additional independent variable representing production rate. The only difference among the models was the manner in which the historical data were aggregated. To insure these distinctions remain clear, each model is briefly restated prior to the presentation of the test results.

Model 1. The results of Model 1 are contained in Table 4. Reduced model:

$$Y_u = B_0 \cdot X_{1a}^{B_1}$$

or in logarithmic form:

$$\log (Y_u) = \log (B_0) + B_1 \cdot \log (X_{1a}).$$

Full model:

$$Y_u = B_0 \cdot X_{1a}^{B_1} \cdot X_{2a}^{B_2}$$

or in logarithmic form:

$$\log (Y_u) = \mathrm{Log}\ (B_0) + B_1 \cdot \log (X_{1a}) + B_2 \cdot \log (X_{2a}).$$

where:

Y_u = unit cost (direct labor hours/equivalent unit/accounting month)

TABLE 4

RESEARCH HYPOTHESIS ONE RESULTS FOR MAGNAVOX MODEL 1

Test Items	Reduced Model	Full Model
Estimated B_0	masked[a]	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	189.33	109.87
F Critical (2,36)	----	3.25
Statistical Hypothesis 1A	----	Reject H_0
F*	----	5.81
F Critical (1,36)	----	4.11
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot[b]	----	Unacceptable
Criterion Test 1A	----	Failed
R^2	.836	.859
R^2 (actual)	.482	.579
Criterion Test 1B	----	Failed

[a]Data are considered proprietary by the manufacturer. Accordingly, numerous table values have been masked in the published version of this thesis. Access to these data can be obtained from the authors upon written approval of Magnavox Government and Industrial Electronics Company.

[b]Residual plots were obtained through the use of Statistical Package for the Social Sciences (SPSS) routines (18:320-367).

X_{1a} = cumulative output plot point adjusted to lot midpoint.

X_{2a} = daily average production rate/accounting month.

Model 2. The results of Model 2 are contained in Table 5. Reduced model:

$$Y_C = B_0 \cdot X_1^{B_1}$$

or in logarithmic form:

$$\log (Y_C) = \log (B_0) + B_1 \cdot \log (X_1).$$

Full model:

$$Y_c = B_0 \cdot X_1^{B_1} \cdot X_{2a}^{B_2}$$

or in logarithmic form:

$$\log (Y_c) = \mathrm{Log} (B_0) + B_1 \cdot \log (X_1) + B_2 \cdot \log (X_{2a}).$$

where:

Y_c = cumulative average cost (cumulative average direct labor hours/equivalent unit).

X_1 = cumulative output plot point (cumulative units at end of accounting month).

X_{2a} = daily average production rate/accounting month.

TABLE 5

RESEARCH HYPOTHESIS ONE RESULTS FOR MAGNAVOX MODEL 2

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	284.80	155.12
F Critical (2,36)	----	3.25
Statistical Hypothesis 1A	----	Reject H_0
F*	----	3.81
F Critical (1,36)	----	4.11
Statistical Hypothesis 1B	----	Fail to Reject H_0
Residual Plot	----	Unacceptable
Criterion Test 1A	----	Failed
R^2	.885	.896
R^2 (actual)	.816	.828
Criterion Test 1B	----	Failed

Model 3. The results of Model 3 are contained in Table 6. Reduced model:

$$Y_u = B_0 \cdot X_{1a}^{B_1}$$

or in logarithmic form:

$$\log (Y_u) = \log (B_0) + B_1 \cdot \log (X_{1a}).$$

Full model:

$$Y_u = B_0 \cdot X_{1a}^{B_1} \cdot X_{2c}^{B_2}$$

or in logarithmic form:

$$\log (Y_u) = \mathrm{Log} (B_0) + B_1 \cdot \log (X_{1a}) + B_2 \cdot \log (X_{2c}).$$

where:

Y_u = unit cost (direct labor hours/equivalent unit/accounting month)

X_{1a} = cumulative output plot point adjusted to lot midpoint.

X_{2c} = cumulative average of the daily average production rates for accounting months to date.

Model 4. The results of Model 4 are contained in Table 7. Reduced model:

$$Y_c = B_0 \cdot X_1^{B_1}$$

TABLE 6

RESEARCH HYPOTHESIS ONE RESULTS FOR MAGNAVOX MODEL 3

Tested Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	189.33	518.33
F Critical (2,36)	----	3.25
Statistical Hypothesis 1A	----	Reject H_0
F*	----	139.36
F Critical (1,36)	----	4.11
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Acceptable
Criterion Test 1A	----	Passed
R^2	.837	.966
R^2 (actual)	.482	.950
Criterion Test 1B	----	Passed

TABLE 7

RESEARCH HYPOTHESIS ONE RESULTS FOR MAGNAVOX MODEL 4

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	284.02	583.13
F Critical (2,36)	----	3.25
Statistical Hypothesis 1A	----	Reject H_0
F*	----	102.23
F Critical (1,36)	----	4.11
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Unacceptable
Criterion Test 1A	----	Failed
R^2	.885	.970
R^2 (actual)	.816	.965
Criterion Test 1B	----	Passed

or in logarithmic form:

$$\log (Y_c) = \log (B_0) + B_1 \cdot \log (X_1).$$

Full model:

$$Y_c = B_0 \cdot X_1^{B_1} \cdot X_{2c}^{B_2}$$

or in logarithmic form:

$$\log (Y_c) = \mathrm{Log} (B_0) + B_1 \cdot \log (X_1) + B_2 \cdot \log (X_{2c}).$$

where:

Y_c = cumulative average cost (cumulative average direct labor hours/equivalent unit)

X_1 = cumulative output plot point (cumulative units at end of accounting month).

X_{2c} = cumulative average of the daily average production rates for accounting months to date.

Model 5. The results of Model 5 are contained in Table 8. Reduced model:

$$Y = B_0 \cdot X_1^{B_1}$$

or in logarithmic form:

$$\log (Y_t) = \log (B_0) + B_1 \cdot \log (X_1).$$

TABLE 8

RESEARCH HYPOTHESIS ONE RESULTS FOR MAGNAVOX MODEL 5

Tested Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	0.02	36.57
F Critical (2,36)	----	3.25
Statistical Hypothesis 1A	----	Reject H_0
F*	----	73.06
F Critical (1,36)	----	4.11
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Acceptable
Criterion Test 1A	----	Passed
R^2	.001	.670
R^2 (actual)	.022	.567
Criterion Test 1B	----	Failed

Full model:

$$Y_t = B_0 \cdot X_{1a}^{B_1} \cdot X_{2a}^{B_2}$$

or in logarithmic form:

$$\log(Y_t) = \text{Log}(B_0) + B_1 \cdot \log(X_1) + B_2 \cdot \log(X_{2m}).$$

where:

Y_t = unit cost (direct labor hours/accounting month).

X_1 = cumulative output plot point (cumulative units at end of accounting month).

X_{2a} = monthly production rate (equivalent units produced/accounting month).

Research Hypothesis One
Analysis Summary

As indicated in Tables 4 through 8, only Magnavox Model 3 was validated for further testing under research hypothesis two. It should also be noted that the additional variation explained by the inclusion of the production rate variable [tested by statistical hypothesis one (b)] was held to be statistically significant at the 0.05 level of significance in all cases except one (Magnavox Model 2). However, even in the case of Magnavox Model 2 rejection of H_0 for statistical hypothesis one (B) would have occurred if tested at the 0.06 level of significance. It is concluded, therefore, that research

hypothesis one was supported by the results obtained through analysis of the Magnavox data.

Analysis of Research Hypothesis Two

Analysis of the predictive ability of Magnavox Model 3 was conducted using the computer program listed in Appendix A. This program contains an option that permits the researcher to perform stepwise truncation of input data points and simulate predictions of direct labor requirements. Predicted values are compared with the observed values and any deviation is computed both as an absolute deviation and as a percentage of the observed value. Since this process is carried out simultaneously for both the full and reduced models, it permits a comparison of the predictive ability of the learning curve with the cumulative production and production rate model.

As an example, if the input data base contains thirty-nine data points (as in the case of the Magnavox data), the last data point (case number 39) is truncated. Regression coefficients are computed for the full and reduced models using thirty-eight data points and these coefficients are used to predict the direct labor requirements for case number 39. The predicted values for the full and reduced models are subtracted from the observed values and the absolute value of the resulting deviations

is stored in an array for use in the test of statistical hypothesis two. The deviation is also divided by the observed value and multiplied by 100 to arrive at a percentage deviation for use in criterion test two.

The above process is repeated for case number 39 using the original data base truncated to 37, 36, ... etc. data points. The stepwise truncation continues until a prediction of case number 39 has been made from data points eighteen months prior to case number 39, and the entire procedure is repeated for cases 38 through 21. In data bases where the data points represent one-month intervals, this procedures result in 324 test situations and provides a subjective test of a model's predictive ability.

An example of the computer printout obtained for case number 39 (the last data point) of the Maganvox Model 3 data base is presented in Figure 1. In addition, a complete computer run, using simulated data, is included with the listing of the program PRODRATE in Appendix A.

For ease of reference and understanding, the statistical hypothesis and criterion test for research hypothesis two are summarized and restated as follows:

<u>Research Hypothesis Two</u>. For eighteen months into the future the predictive ability of the full model is better that the predictive ability of the reduced model.

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 39 WHICH HAS AN OBSERVED VALUE OF: MASKED

# CASES USED	REDUCED (LEARNING CURVE) MODEL				FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL				
	PREDICTION	% DEVIATION	EST B0	EST B1	PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
38	MASKED	-10.91	MASKED	MASKED	MASKED	-0.28	MASKED	MASKED	MASKED
37	------	-11.35	------	------	------	0.01	------	------	------
36	------	-12.04	------	------	------	0.24	------	------	------
35	------	-13.52	------	------	------	-0.27	------	------	------
34	------	-14.53	------	------	------	-0.30	------	------	------
33	------	-15.15	------	------	------	0.20	------	------	------
32	------	-16.23	------	------	------	0.59	------	------	------
31	------	-17.42	------	------	------	0.95	------	------	------
30	------	-19.09	------	------	------	0.95	------	------	------
29	------	-20.63	------	------	------	1.47	------	------	------
28	------	-23.09	------	------	------	1.30	------	------	------
27	------	-26.00	------	------	------	1.05	------	------	------
26	------	-28.70	------	------	------	1.46	------	------	------
25	------	-31.86	------	------	------	1.62	------	------	------
24	------	-35.15	------	------	------	1.96	------	------	------
23	------	-38.58	------	------	------	2.32	------	------	------
22	------	-42.71	------	------	------	2.31	------	------	------
21	------	-49.15	------	------	------	-0.25	------	------	------

Fig. 1. Predictive Ability Test Results for Magnavox Case Number 39

NOTE: Masked denotes proprietary data.

Statistical Hypothesis Two. H_0: $|\bar{D}_R| \leq |\bar{D}_F|$; H_a: $|\bar{D}_R| > |\bar{D}_F|$. Reject H_0 if $Z > Z_c$ (.05).

Criterion Test Two. The model with the better practical predictive ability is the model whose percent deviations more frequently fall in the excellent (5 percent) or good (10 percent) categories over the range of all test situations.

Research Hypothesis Two Analysis Summary

A summary of the predictive ability tests conducted for research hypothesis two on Model 3 of the Magnavox data is contained in Table 9. These results clearly indicate that the full model was a better predictor of direct labor requirements than was the reduced model. Not only was the full model's predicted value consistently closer to the observed value, but the full model's predictive ability showed more stability over the range of data tested. It would naturally be expected that the predictions made by both models would gradually become less accurate as the number of data points used in formulating the regression coefficients decreased. This trend was observed in the predictive ability test for both models, but the full model demonstrated much less loss of precision as data points were decreased than did the reduced model. In addition, the reduced model was observed to consistently overestimate the direct labor requirements in every

TABLE 9

RESEARCH HYPOTHESIS TWO RESULTS FOR MAGNAVOX MODEL 3

Test Items	Reduced Model	Full Model
Average absolute deviation	37.63	5.30
Variance	605.34	43.61
Z Test Statistics	----	22.85
Z critical	----	1.65
Statistical Hypothesis Two	----	Reject H_0
Total number of test situations	324	324
Total number of excellent predictions (within 5 percent)	0	159
Total number of good predictions (within 10 percent)	5	250
Criterion Test Two	----	Passed

test situation, while no such trend was observed for the full model.

In summary, therefore, research hypothesis two was considered supported by the results obtained in the predictive ability tests conducted on the Magnavox data.

The Teledyne Data

The data provided by the Teledyne Systems Company consisted of production direct labor requirements for the Computer Signal Data Converter (CSDC). Made up of forty-seven data points, the data represented accounting months from January 1975, when full-scale production commenced, through November 1978. Like the Magnavox data, the Teledyne data pertained to an ongoing program and thus were good candidates for this research (8:97). In addition, the production rate of the CSDC had been constrained since December 1977 by a government slowdown in the rate of delivery of these units. This additional fact enhanced the test situation for observing the effect of an exogenous change in production rate on direct labor requirements.

The raw CSDC production data were treated as described in the explanation of individual variables for each of the models presented in Chapter III. Regression analysis technique, statistical hypothesis testing, and criterion testing for research hypotheses one and two were

performed in exactly the same manner as performed on the Magnavox data.

Analysis of Research Hypothesis One

Once again, to insure the distinctions among models remain clear, each model is briefly restated prior to the tabular presentation of the test results.

Model 1. The results of Model 1 are contained in Table 10. Reduced model:

$$Y_u = B_0 \cdot X_{1a}^{B_1}$$

or in logarithmic form:

$$\log (Y_u) = \log (B_0) + B_1 \cdot \log (X_{1a}).$$

Full model:

$$Y_u = B_0 \cdot X_{1a}^{B_1} \cdot X_{2a}^{B_2}$$

or in logarithmic form:

$$\log (Y_u) = \mathrm{Log}\ (B_0) + B_1 \cdot \log (X_{1a}) + B_2 \cdot \log (X_{2a}).$$

where:

Y_u = unit cost (direct labor hours/equivalent unit/accounting month)

X_{1a} = cumulative output plot point adjusted to lot midpoint.

TABLE 10

RESEARCH HYPOTHESIS ONE RESULTS FOR TELEDYNE MODEL 1

Tested Items	Reduced Model	Full Model
Estimated B_0	masked[a]	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	3.39	16.85
F Critical (2,44)	----	3.21
Statistical Hypothesis 1A	----	Reject H_0
F*	----	28.25
F Critical (1,44)	----	4.06
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot[b]	----	Acceptable
Criterion Test 1	----	Passed
R^2	.070	.434
R^2 (actual)	.073	.433
Criterion Test 2	----	Failed

[a]Data are considered proprietary by the manufacturer. Accordingly, numerous table values have been masked in the published version of this thesis. Access to these data can be obtained from the authors upon written approval of the Teledyne Systems Company.

[b]Residual plots were obtained through the use of Statistical Package for the Social Sciences (SPSS) routines (18:320-367).

X_{2a} = daily average production rate/accounting month.

<u>Model 2</u>. The results of Model 2 are contained in Table 11. Reduced model:

$$Y_C = B_0 \cdot X_1^{B_1}$$

or in logarithmic form:

$$\log (Y_C) = \log (B_0) + B_1 \cdot \log (X_1).$$

Full model:

$$Y_c = B_0 \cdot X_1^{B_1} \cdot X_{2a}^{B_2}$$

or in logarithmic form:

$$\log (Y_c) = \mathrm{Log}\ (B_0) + B_1 \cdot \log (X_1) + B_2 \cdot \log (X_{2a}).$$

where:

Y_c = cumulative average cost (cumulative average direct labor hours/equivalent unit).

X_1 = cumulative output plot point (cumulative units at end of accounting month).

X_{2a} = daily average production rate/accounting month.

<u>Model 3</u>. The results of Model 3 are contained in Table 12. Reduced model:

TABLE 11

RESEARCH HYPOTHESIS ONE RESULTS FOR TELEDYNE MODEL 2

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	940.85	545.55
F Critical (2,44)	----	3.21
Statistical Hypothesis 1A	----	Reject H_0
F*	----	7.81
F Critical (1,44)	----	4.06
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Unacceptable
Criterion Test 1	----	Failed
R^2	.954	.961
R^2 (actual)	.955	.962
Criterion Test 2	----	Passed

TABLE 12

RESEARCH HYPOTHESIS ONE RESULTS FOR TELEDYNE MODEL 3

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	3.39	21.85
F Critical (2,44)	----	3.21
Statistical Hypothesis 1A	----	Reject H_0
F*	----	37.55
F Critical (1,44)	----	4.06
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Acceptable
Criterion Test 1	----	Passed
R^2	.070	.498
R^2 (actual)	.073	.502
Criterion Test 2	----	Failed

$$Y_u = B_0 \cdot X_{1a}^{B_1}$$

or in logarithmic form:

$$\log (Y_u) = \log (B_0) + B_1 \cdot \log (X_{1a}).$$

Full model:

$$Y_u = B_0 \cdot X_{1a}^{B_1} \cdot X_{2c}^{B_2}$$

or in logarithmic form:

$$\log (Y_u) = \text{Log } (B_0) + B_1 \cdot \log (X_{1a}) + B_2 \cdot \log (X_{2c}).$$

where:

Y_u = unit cost (direct labor hours/equivalent unit/accounting month)

X_{1a} = cumulative output plot point adjusted to lot midpoint.

X_{2c} = cumulative average of the daily average production rates for accounting months to date.

<u>Model 4</u>. The results of Model 4 are contained in Table 13. Reduced model:

$$Y_c = B_0 \cdot X_1^{B_1}$$

or in logarithmic form:

$$\log (Y_c) = \log (B_0) + B_1 \cdot \log (X_1).$$

TABLE 13

RESEARCH HYPOTHESIS ONE RESULTS FOR TELEDYNE MODEL 4

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	940.85	681.88
F Critical (2,44)	----	3.21
Statistical Hypothesis 1A	----	Reject H_0
F*	----	20.26
F Critical (1,44)	----	4.06
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Unacceptable
Criterion Test 1	----	Failed
R^2	.954	.969
R^2 (actual)	.955	.968
Criterion Test 2	----	Passed

Full model:

$$Y_c = B_0 \cdot X_1^{B_1} \cdot X_{2c}^{B_2}$$

or in logarithmic form:

$$\log (Y_c) = \text{Log } (B_0) + B_1 \cdot \log (X_1) + B_2 \cdot \log (X_{2c}).$$

where:

Y_c = cumulative average cost (cumulative average direct labor hours/equivalent unit)

X_1 = cumulative output plot point (cumulative units at end of accounting month).

X_{2c} = cumulative average of the daily average production rates for accounting months to date.

Model 5. The results of Model 5 are contained in Table 14. Reduced model:

$$Y_t = B_0 \cdot X_1^{B_1}$$

or in logarithmic form:

$$\log (Y_t) = \log (B_0) + B_1 \cdot \log (X_1).$$

Full model:

$$Y_t = B_0 \cdot X_1^{B_1} \cdot X_{2m}^{B_2}$$

or in logarithmic form:

$$\log (Y_t) = \text{Log } (B_0) + B_1 \cdot \log (X_1) + B_2 \cdot \log (X_{2m}).$$

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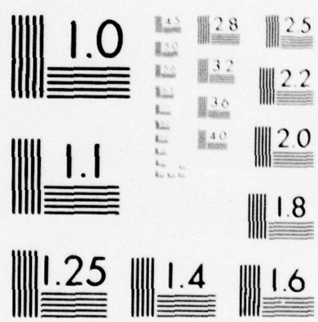

MICROCOPY RESOLUTION TEST CHART

TABLE 14

RESEARCH HYPOTHESIS ONE RESULTS FOR TELEDYNE MODEL 5

Test Items	Reduced Model	Full Model
Estimated B_0	masked	masked
Estimated B_1	masked	masked
Estimated B_2	----	masked
F Ratio	17.45	1,330.95
F Critical (2,44)	----	3.21
Statistical Hypothesis 1A	----	Reject H_0
F*	----	1,905.79
F Critical (1,44)	----	4.06
Statistical Hypothesis 1B	----	Reject H_0
Residual Plot	----	Acceptable
Criterion Test 1	----	Passed
R^2	.279	.984
R^2 (actual)	.235	.981
Criterion Test 2	----	Passed

where:

Y_t = total cost (total direct labor hours/accounting month).

X_1 = cumulative output plot points (cumulative units at end of accounting month).

X_{2m} = monthly production rate (equivalent units produced).

Research Hypothesis One
Analysis Summary

As indicated in Tables 10 through 14, only Teledyne Model 5 was validated for further testing under research hypothesis two. As in the case of the Magnavox data, regression analysis of the Teledyne data indicated the addition of the production rate variable added significantly to the explanation of variations in the direct labor hour requirements. In fact, the results of testing for statistical hypothesis one (B) in each model demonstrated that the explanatory power added by the production rate variable was statistically significant at the 0.05 level of significance in all models. These results support the validity of research hypothesis one for the Teledyne data.

Model 5 was then tested for predictive ability under research hypothesis two.

Analysis of Research Hypothesis Two

Analysis of the predictive ability of Teledyne Model 5 was conducted in exactly the same manner as described for the Magnavox data. An example of the computer printout for case number 47 (the last data point) of the Teledyne data is presented in Figure 2.

Research Hypothesis Two Analysis Summary

A summary of the predictive ability tests conducted for research hypothesis two on Model 5 of the Teledyne data is contained in Table 15. These results demonstrate that the full model was a better predictor of direct labor requirements than was the reduced model. Once again, as originally observed in the Magnavox Model 3 testing, the reduced model consistently overestimated the direct labor hours variable in the majority of the situations.

In summary, therefore, research hypothesis two was considered supported by the results obtained in the predictive ability tests conducted on the Teledyne data.

Differences, Similarities and Summary

Of the ten models investigated (five for each data set), there were obvious differences and similarities between the Magnavox data and the Teledyne data. First, the differences will be contrasted; then the similarities will be addressed. Finally, a summary of the research findings

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 47 WHICH HAS AN OBSERVED VALUE OF: MASKED

# CASES USED	REDUCED (LEARNING CURVE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
46	MASKED	-40.67	MASKED	MASKED	MASKED	2.17	MASKED	MASKED	MASKED
45	------	-41.26	------	------	------	2.91	------	------	------
44	------	-43.49	------	------	------	3.12	------	------	------
43	------	-45.75	------	------	------	3.33	------	------	------
42	------	-46.60	------	------	------	3.47	------	------	------
41	------	-49.16	------	------	------	3.76	------	------	------
40	------	-52.54	------	------	------	3.98	------	------	------
39	------	-53.52	------	------	------	3.93	------	------	------
38	------	-56.81	------	------	------	4.22	------	------	------
37	------	-60.67	------	------	------	4.70	------	------	------
36	------	-62.61	------	------	------	6.47	------	------	------
35	------	-67.61	------	------	------	7.65	------	------	------
34	------	-71.40	------	------	------	8.55	------	------	------
33	------	-73.43	------	------	------	8.00	------	------	------
32	------	-72.58	------	------	------	8.90	------	------	------
31	------	-74.95	------	------	------	8.20	------	------	------
30	------	-73.16	------	------	------	8.30	------	------	------
29	------	-75.71	------	------	------	7.33	------	------	------

Fig. 2. Predictive Ability Test Results for Teledyne Case Number 47.

NOTE: Masked denotes proprietary data.

TABLE 15

RESEARCH HYPOTHESIS TWO RESULTS FOR
TELEDYNE MODEL 5

Test Items	Reduced Model	Full Model
Average absolute deviation	1,691.46	195.16
Variance	627,855.45	17,587.80
Z Test Statistics	----	33.52
Z critical	----	1.65
Statistical Hypothesis Two	----	Reject H_0
Total number of test situations	324	324
Total number of excellent predictions (within 5 percent)	15	227
Total number of good predictions (within 10 percent)	35	315
Criterion Test Two	----	Passed

and an overall evaluation of the applicability of the cumulative production and production rate model to the two data sets will close Chapter IV.

<u>Differences</u>

The reader is probably struck first, as the researchers were, by the fact that the best predictive model was not the same model for the two data sets. Upon reflection, however, this difference was not a surprise because each individual firm can be as different as are various products or industries. For the Magnavox data, the best predictor was Model 3, whose dependent variable was average direct labor hours per equivalent unit per accounting month. The X_2 independent variable in Model 3 for the Magnavox data was represented by the cumulative average of the daily average production rate. For the Teledyne data, however, the best predictor was Model 5 whose dependent and X_2 independent variables were total labor hours per accounting month and monthly production rate, respectively.

The researchers concluded the difference was accounted for by the dissimilar environments of the two production programs. For example, the Magnavox production was characterized by direct labor operations which were machine-paced under the concept of constant work force. Conversely, the Teledyne production was labor-intensive and constrained by government-imposed production rates

which were much lower than those of Magnavox. As a result, for Magnavox, as the average direct labor hours decreased, the production rate rose as a direct consequence of the constant work force. Thus, for Magnavox the production rate was very sensitive, exhibited a wide range, and generally increased steadily. Conversely, for Teledyne, as the direct labor hours required per equivalent unit decreased, personnel were reassigned to other tasks so that the constrained production rate would not be exceeded. In sum, the production rate would be expected to have different influences in the two different situations.

Given the differences, the logic of Model 3 for Magnavox is listed. To a point, the production rate at Magnavox generally increased steadily as the program progressed; likewise, to a point the average direct labor hours required per equivalent unit generally decreased steadily. Therefore, the unit cost curve would be expected to be more predictive than the cumulative average curve which smooths the dependent variable. Moreover, the smoothed production rate variable of Model 3, when added to the presence of cumulative output, served to smooth the relatively large changes in the observed hours per unit. In short, the unit cost curve was adjusted for midpoints to approach the actual average cost of the lots; once these data were augmented by a smoothed production rate

variable, the lagged effect (i.e., the cumulative average rate was always smaller than the current monthly rate) was just enough to explain the relationship unique to Magnavox.

For Teledyne, due primarily to the production rate constraint set by the government and the labor-intensive nature of operation, expectations dictated something entirely different. Unlike Magnavox, the Teledyne program was subject to fluctuating production rates, and not limited to a generally steady movement in one direction. These fluctuations resulted from a combination of factors, to include comparatively few units produced per month due to government constraints and intermittent worker transfer, with its resultant interruptions of workers' continual familiarity with tasks. In sum, a model such as Model 5, whose dependent and X_2 independent variables represented total monthly direct labor hours and monthly production rate, was expected to predict well in an operating environment like that of Teledyne (13:37).

Similarities

Two distinct similarities were noted between the Magnavox and Teledyne avionics production programs. The first concerned preproduction planning and its reflection during data analysis. The second similarity revolved around judgment of the appropriateness of the model being

evaluated, i.e., the analysis of the residual plots during the first criterion test under research hypothesis one.

Preproduction Planning. Both Magnavox and Teledyne planned extensively prior to commencing full-scale production of their respective avionics products. Though decidedly different in technology and approach due to the inherently different backgrounds of the products, each effort was obvious.

The painstaking efforts of Magnavox in design planning were discussed in detail (2:29-37) by Mr. William H. Boden, Program Director, UHF Radios, of Magnavox. In the referenced article, Mr. Boden discussed life cycle cost in terms of production design. Extensive preproduction design and planning generally results in a more stable product design throughout the production phase by reducing the number and severity of engineering change proposals. Thus, a great deal of learning takes place prior to the beginning of full scale production. The effect of this preproduction learning was reflected in the Magnavox data by the character of the rate of learning, as viewed across a thirty-nine month interval.

When preproduction planning is extensive, a cumulative average cost curve is generally more representative, at least during the initial units because the planning serves to reduce the initial amount of learning. Chase and

Aquilano (4:532) had the following to say about preproduction versus postproduction adjustments.

> The amount of learning shown by the learning curve depends both on the initial unit(s) of output and on the learning percentage. If there is much preproduction planning, experimentation, and adjustment, the early units will be produced more rapidily than if improvements are made after the first few units--other things being equal. In the first case, therefore, the apparent learning will be less than in the second case, even though subsequent "actual" learning may be the same in each instance.

Thus, the Magnavox situation was explained as follows. The preproduction efforts were such that, although a model containing the cumulative average cost variable was not the best predictor, the unit cost curve had to be augmented by a smoothed production rate variable to approach accurate cost prediction.

The effect of the timing of adjustments just discussed was also clearly discernable in the Teledyne program. Although Teledyne used engineering development prior to commencing full-scale production, compared to Magnavox the rate of learning was less in the Teledyne program. As earlier implied, this was attributable in part to the combined effects of the government-imposed constraints on the production rate and the labor-intensive operations.

The rate of learning at Teledyne was also partially attributable to the initial units of output, a criterion just cited, and the "unconstant" work force. Contrasted with Magnavox, the initial units of output from

Teledyne were only a fraction as large. Moreover, as learning occurred and the production rate improved, it approached the production constraint. To avoid exceeding the constraint, personnel were moved to different tasks. The result was the "unconstant" work force, which decidedly affected the learning at Teledyne. Specifically, according to Chase and Aquilano (4:532), "We would expect, for example, that more supervisors, repairmen and materiel handlers would speed up production whereas a reduction in their numbers would slow it down." Cheney (5:40) also cited upward trends in costs (or toe-ups) which resulted from the transfer of experienced people to different jobs.

The common denominator of the preceding discussion was captured by Cheney (5:43) as follows:

> In summary, the most important requirement in using learning curves as tools for production cost estimating (or for any other purpose) is that the estimator be thoroughly familiar not only with learning theory, but also with all pertinent aspects of the particular industry to which he is applying learning theory.

To this should be added that the estimator must also be constantly aware of variations within a particular industry--both firms and products.

Analysis of Residuals. Common to both data sets, of course, was the subjective analysis of residuals. Precautions were taken not to overlook any implications of these analyses. Such oversight is not uncommon, according

to Spurr and Bonini (22:607), when computer analysis packages are becoming more and more numerous.

For some models, the patterns in the residuals were clearly unacceptable, e.g., obvious heteroscedasticity. In still other analyses the subjective calls were close. Several possible reasons were considered for the unacceptable patterns in residuals. Those reasons include inadequate collection and recording of production data, as described by Cochran (6:435-437) and Johnson (13:34). Other possible reasons were management control and self-fulfilling prophecy (9:17-23), and the possibility of linear regression not being adequately representative (5:56).

The most likely reason for the patterns was concluded to be within the margin between linearity and the best possible model, i.e., some quadratic regression. The issue of linearity was cited early in this paper as being beyond the scope of this research. It remained beyond the scope, although the authors were compelled to make some observations. According to Cheney (5:56), the issue of linearity is no longer considered important because learning curves are only approximations. Cheney (5:56) also reported Asher's opinion of 1956 that in each specific case it is judgment that determines whether or not a linear curve is appropriate. Beyond that, in the authors' opinions, the judgment, in turn, must necessarily be based

on, or at least be influenced by, the purpose of the research and the inherent tradeoffs.

Cheney (5:56) cited Barrett, whose 1969 Ph.D. dissertation was an empirical study of learning curves which led him to the following two conclusions. First, generally there was one or more nonlinear forms which better fit the data set than did the linear form. Second, there was for each user or investigator a tradeoff between the sacrificed accuracy of simple linearity and the cost of increased accuracy associated with hypotheses about nonlinearity.

Due to the purpose of this research and the inherent tradeoffs, some avenues were not comprehensively explored. Examples of these avenues were as follows:

1. Displacement of autocorrelation schemes, as discussed by Neter and Wasserman (17:352-366). Autocorrelation probably existed to some degree in all models tested, especially in Model 5 which was taken from Johnson who reported autocorrelation (13:38-39). Consequently, while Teledyne Model 5 passed all the tests and was excellent or good in predictive ability, the standard error of the estimate may well be low (13:38). The issue of autocorrelation assumes even greater importance for those who, like Johnson (13:38), view the cumulative output variable as a proxy for time.

2. Variance-stabilizing transformations, as discussed by Neter and Wasserman (17:131-136). All models which were rejected during analysis of the residuals [Criterion Test One (A)] were found unacceptable due to obvious heteroscedasticity. Each rejection was based on careful analysis, to include comparisons with Neter and Wasserman's prototype residual plots (17:101).

3. F test for linearity (17:113-121). Each rejection based on subjective evaluation of residuals can be tested statistically through the F test for linearity.

In conclusion, there was a tradeoff in this research between improved but unneeded accuracy and the convenience of linearity. The subjective analysis of residuals caused rejection of several models due to inappropriateness. Those models which were acceptable, however, were distinguished by two features. First, they had already passed all statistical tests under research hypothesis one. Second, they also included models whose predictive abilities were good enough under research hypothesis two to meet the objectives of this research.

Summary

The preceding analyses of the two avionics production programs were based on the methodology and treatment of data as earlier explained in Chapter III. For both data sets, Magnavox and Teledyne, research hypothesis one was accepted. Analysis of the data revealed that

production rate was an important explainer of variation in direct labor hours for nine of the ten models evaluated. Had the statistical tests been at the 0.06 level of significance, all ten models would have confirmed production rate as an important explainer of the variation. Moreover, in seven out of ten models tested under research hypothesis one, the increase in R^2 (actual) was 9 percent or more once cumulative output was augmented by the production rate; in half of the models the increase in R^2 actual was 30 percent or more. The summary statistics for R^2 (actual) are found in Table 16.

For both data sets, Magnavox and Teledyne, research hypothesis two was accepted. Not only was the increased predictive ability of the full model statistically higher when tested at the 0.05 level of significance, but the subjective tests also clearly demonstrated the superiority of the full model.

For the overwhelming majority of all test situations, the inclusion of the production rate variable served to stabilize the predictions when they were over a long time span. This finding supported Congleton and Kinton's research (8:83), as well as Smith's findings (21:136). In other words, the full model not only predicted better over the eighteen month time span, but its predictions were also far less susceptible to loss of precision as data points were truncated. Still another

TABLE 16

INCREASES IN R^2 (ACTUAL) FOR ALL TEN MODELS TESTED AFTER INCLUSION OF PRODUCTION RATE

Model	R^2 Reduced	R^2 Full	ΔR^2
Magnavox 1	48.0	57.0	9.0
Magnavox 2	81.6	82.8	1.2
Magnavox 3	48.2	95.0	46.8
Magnavox 4	81.6	96.5	14.9
Magnavox 5	2.2	56.7	54.5
Teledyne 1	7.3	43.3	36.0
Teledyne 2	95.5	96.2	0.7
Teledyne 3	7.3	50.2	42.9
Teledyne 4	95.5	96.8	1.3
Teledyne 5	23.5	98.1	74.6

finding was that the trend observed in the reduced model to consistently overestimate the required direct labor hours was eliminated once production rate was added to the reduced model.

Another observation concerned the sensitivity of the regression coefficients to change as observations were truncated and new regression coefficients were computed, as described in Chapter IV. These results indicated that even within a particular production program, the coefficients were sensitive. This sensitivity suggests that the development of general cost models using coefficients derived from several avionics production programs is inappropriate.

Both the statistical and subjective research findings verified the applicability of the cumulative production and production rate model to the two subject avionics production programs. The potential overall application of this model provokes even more interest when one recognizes the vast difference in the environments of the two subject programs, as described at the beginning of section three of this chapter. That interest is provoked despite the inability to generalize the findings beyond the Magnavox and Teledyne data.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Since the late 1960s cost growth has become an increasingly stronger concern of the Air Force community. Nowhere has this strengthened concern become more visible than in the acquisition sector, especially in connection with the acquisition of major systems. Avionics components are often a part of a major system where costs grow rapidly; therefore, cost estimating assumes great importance.

A key cost element is that of direct labor. Because other costs, to include an overhead charge, are pyramided on it, the direct labor cost is a crucial element is cost estimating. Costs for direct labor are frequently estimated by use of a learning curve model.

Literature Review

Learning curve models have served as techniques for estimating direct labor costs for at least fifty years, but they do not systematically consider the effects of production rate on direct labor costs. In fact, learning curve models do not consider production rate at all, much less the exogenous changes in that rate.

Yet the exogenous changes do occur, as evidenced in this research by the government imposed constraints on Teledyne. Failing to consider the production rate changes does not make sense. For example, as Johnson (13:25) noted, the learning curve model would predict the same labor hour requirements for 1,000 widgets, regardless of whether the production rate was 100 per month or 10 per month.

Investigations of the effects of introduction of the production rate variable into a learning curve model have a long, controversial history. Some investigators have rejected the significance of the production rate. Such rejection has usually been at least partially based on statistical testing. Other investigators have demonstrated that the production rate is in fact a significant explainer. During the past ten years, these latter investigations have resulted in compatible findings about the production rate in airframe manufacturing. It was the compatibility of these findings attesting to the significance of production rate in airframe manufacturing that formed the basis for this research. Based on Smith's model, this research extended the investigation of production rate from airframes to avionics.

The Model

Smith's model was developed several years ago and later replicated by Congleton and Kinton. It is an adaptation of Orsini's model and the full model is as follows:

$$Y = B_0 \cdot X_1^{B_1} \cdot X_2^{B_2} \cdot 10^e$$

where, in general terms, the variables are defined as follows:

Y represents the direct labor hours,

X_1 represents the cumulative output variable,

and

X_2 represents the production rate variable.

Transformed to logarithmic terms, the full model becomes:

$$\text{Log } Y = \text{Log } B_0 + B_1 \text{ Log } X_1 + B_2 \text{ Log } X_2 + e.$$

The reduced model is identical to the full model, before and after transformation, except the reduced model does not include the second independent variable, production rate. In other words, the reduced model is a learning curve model. Specific arrangements of variables and treatment of the raw data sets in this research were discussed in Chapter III.

Research Objectives

This research had two research objectives. The first was to determine if there was an effect by production rate changes on production direct labor requirements for avionics production. The second objective was to evaluate the predictive ability of the full model, once it was tailored to a particular avionics production program, for eighteen months into the future.

Methodology

The methodology for this research was centered on multiple linear regression analysis, given transformation of the model, just explained, and treatment of the data sets. Several steps comprised the total requirements a model had to meet to transit through the entire methodology.

Research hypothesis one stated that production rate was an important explainer of direct labor hours when included in an appropriate model. First, the F test was used to check for a relationship of the dependent variable, direct labor hours, to the set of independent variables, cumulative output and production rate. This check constituted statistical test one (A). Statistical test one (B), on the other hand, was used to test the production rate (B_2) coefficient for significance.

Following the statistical tests, two criterion tests were used under the first research hypothesis. The

first criterion test was used to evaluate and accept or fail to accept the residuals from regression analysis. The second criterion test required both R^2 and R^2 (actual) to exceed 85 percent, an arbitrary figure based on the experience of the authors. All models passing both statistical tests and both criterion tests were then tested under research hypothesis two.

Research hypothesis two was that for eighteen months into the future the full model was a better predictor than the reduced model. The comparative predictive abilities of the models, full and reduced, were evaluated both statistically and subjectively against the observed values. This evaluation was made possible by using the same truncation and simulation procedure that Smith used. Statistical test two then compared the average absolute deviations of the full versus reduced models over an eighteen month interval. The subjective test, on the other hand, compared predictions of both models, full and reduced, with the observed values and the deviations in this case were classified into categories of good and excellent. Thus, the criterion test two results from the subjective comparisons lended a measure of practicability to the research.

Excepting analysis of residuals, all test results were achieved through use of the computer program PRODRATE which is listed and described in Appendix A. PRODRATE is a modified version of the Fortran IV program used by

Smith. A sample computer run of the entire PRODRATE program is also included in Appendix A. The data set for this sample is an artifcial set and both selection options are illustrated.

Conclusions

The objectives of this research were achieved. Production rate was found to be a significant explainer of variation in direct labor hours in nine of ten cases. Had the statistical tests been at the 0.06 level of significance, the test cases would have unanimously supported the first research hypothesis. The predictive ability of the full model was better than that of the reduced model for eighteen months into the future. That superiority was supported statistically by measuring the average absolute deviations over an eighteen month interval in which 324 test situations were evaluated. Superiority was also supported in subjective tests.

In addition to the research objectives, per se, leading to the two prime conclusions, several additional conclusions resulted from this research. Each of these additional conclusions will be discussed in turn as they appear; then corollary findings and recommendations will close this paper.

First Additional Conclusion. The addition of the production rate variable to the learning curve model had

desirable effects other than those directly anticipated as results of the two research objectives. The two which followed from the research objectives were just listed. Other conclusions were reached, beginning with what might be termed "stability."

Whereas the learning curve model consistently overestimated the direct labor hours requirements, the full model did not. Thus, just as Smith found, this research found that the production rate, when included in an appropriate model, stabilized the predictions over an extended interval. This finding alone, without regard to statistical tests of explained variation and predictive ability, indicates that inclusion of production rate (at least in the programs evaluated by this research) leads to more accurate projections of direct labor hours requirements.

Second Additional Conclusion. The results of this research clearly indicate that the objectives were met; however, at the same time, another obvious conclusion is that the regression coefficients are unique to the program for which they were derived. This uniqueness holds for the Magnavox coefficients for that program, and it holds for the Teledyne coefficients for the Teledyne program. That obvious conclusion was consistently evidenced during simulations of predicting the future, using the truncation method described in Chapter III. During those truncations,

the coefficients were sometimes very sensitive to change during a single iteration (e.g., going from a 28-point data base to a 27-point data base, as explained in Chapter IV under analysis of research hypothesis two for the Magnavox data). Thus, generalizing to other programs cannot work; even for the same program, month-to-month sensitivity can be large.

Third Additional Conclusion. Despite the inability to generalize coefficients, the overall applicability of the cumulative production and production rate model appears to have wide potential. In this research the model was shown to apply equally well as an alternative to the learning curve model in predicting direct labor hours for two decidedly different production programs. On the one hand, the model was tailored to a machine-paced program which operated under the concept of constant work force and whose initial units were moderate to large in number. Conversely, in the other case, the model was also tailored to a production program which was labor intensive and whose production rates were constrained by government imposed limits. Its initial units of production were low in number. The point is simply this: If the model can be tailored to such diverse programs, its potential elsewhere and in between practically begs for attention. Smith's model has been validated within avionics production programs.

Corollary Findings. Several corollary findings were considered for inclusion in this research. Along the way it was very tempting to go off on tangents and investigate some other interesting topics. Examples included the issues associated with residual analysis (e.g., autocorrelation and homoscedasticity), multicollinearity, patterns of regression coefficients, cumulative average versus unit curves, learning curve versus the experience or improvement curve, and nonlinear regression analysis. As the research progressed, it became evident that there were limits to what could be achieved. Consequently, only two corollary findings were written. Listed as PRODRATE and multicollinearity, in the appendixes to this research, the two corollary findings were included with the intent of providing findings which would be of the most practical value to the readers, the price analysts, and future researchers.

Recommendations

The most obvious recommendation is listed first. Not only can Smith's model be used in the ongoing programs which were evaluated during this research, but the model should be tested within other ongoing programs of avionics production.

All sorts of possibilities are open in avionics for the aggressive researchers who want to link production rate to other cost elements. For example, the level of

aggregation of costs (comparable to Smith's fabrication, assembly, and total hours in airframes) in avionics appears untouched. Another interesting twist, alluded to in Chapter III, is the direct labor versus total costs. Still another approach could address the direct manufacturing labor hours versus all direct labor, as discussed by Johnson (13:26).

In final conclusion, then, the cumulative production and production rate model as developed by Smith appears worthy of even further application within avionics production and attempted extensions elsewhere. The authors believe that future researchers have an area open to them limited only by their ingenuity in recombining the raw data of a particular production program.

APPENDIXES

APPENDIX A

THE COMPUTER PROGRAM PRODRATE

This appendix is made up of three sections. The first section describes how the program works and what it does. The second section lists the computer program PRODRATE in its entirety. Finally, there is a sample output, to include both selective options described in section one of this appendix. The output in section three comes from an artificial data base.

A Description of the Computer Program PRODRATE

Upon beginning this research, the authors were at first overwhelmed at the variety and complexity of the computer programs available for multiple linear regression analysis. The majority of these programs converted the input variables to logarithms prior to regressions. A few computer programs were available which conducted nonlinear regression directly through an iterative process. None of the available programs were ideally suited for this research, however, because the authors wished to look at the predictive ability of the full and reduced models in addition to the regression results.

The computer program written by Smith (21:147-153) for his research came very close to meeting the needs of the authors; but some modifications were necessary to provide for the predictive ability tests. In addition,

Smith's program had been loaded into the COPPER IMPACT computer system[14] but had not been debugged and made operational. Thus, with Smith's permission, the authors decided to modify his program, rather than construct an entirely new program.

The primary objective of this modification was to make the interactive program available on COPPER IMPACT as an additional tool for cost comparison. A secondary objective was to format the computer output to make it easy to read and facilitate comparisons between the full and reduced models. The program PRODRATE, listed in this appendix, was the result of the modification.

To make this program accessible to the government cost and price analysts, PRODRATE has been included in the COPPER IMPACT library.

The program reads the input data from any file specified by the user. Instructions on how to build a data file are available in the program. This feature allows the user to change the form of the model (e.g., unit curve, cumulative curve, total cost curve) simply by

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[14]COPPER IMPACT is the project name of an aggressive program to Improve Modern Pricing And Costing Techniques used in the Air Force contracting process. This program emphasizes the implementation of advanced computer techniques applied to price and cost analysis and the training of Air Force personnel in the skillful use of these techniques (25:1). The time-sharing computer system is designated by the same name, COPPER IMPACT, and is government-leased, currently from General Electric.

making the necessary modifications to the data base. In addition, an option within the program allows the user to list the input data as they were read from the data file and converted to logarithms.

Analysis of the data is begun by calculating and printing the Pearson correlation coefficients of the three variables: direct labor requirements, cumulative production, and production rate. Log-linear regression is first performed between the direct labor requirements (dependent) variable and the cumulative production (independent) variable. Finally the dependent variable is regressed against both independent variables simultaneously.

In presenting the results of each of these three regressions, the program prints a listing of the actual direct labor requirements, the predicted direct labor requirements, and the residuals. This feature of PRODRATE is one the authors found unavailable elsewhere. The obvious advantage is that the user can relate the results to the original untransformed variables (rational numbers, not logarithms) and see how well the untransformed data fit the model.

Following the listing of the residuals, summary statistics are printed for each model. They include the values for the coefficients (exponents), standard errors, F ratios, R^2, R^2 (actual), and learning factor. Two

selective options for which additional printouts are available are the Predictive Ability Tests and the Projection and Sensitivity Matrix. For a quick-look analysis of several different models, the program can be preset to stop after three regression analyses by not selecting the additional options.

The Predictive Ability Test option permits the user to select the number of data points (cases) to be truncated during the test and thus, control the time span over which the test is conducted. The test is performed using nested DO loops to perform a stepwise truncation of the data points. The truncated data is then predicted using the regression results of the remaining data. After all truncated cases have been predicted and results printed, a summary table is printed. This summary table contains data on statistical significance and permits subjective comparisons of the accuracy of predictions made by the full and reduced models. This process is described in Chapter IV just prior to the presentation of predictive ability test results for the Magnavox Model 3. In addition, an example is provided in section three of this appendix.

The Projection and Sensitivity Matrix option of program PRODRATE provides the cost and price analyst with a means of predicting direct labor requirements at varying production rates. This option also permits the user

to see the sensitivity of the direct labor requirements to changes in production rate over a wide range of cumulative production. The last observed values for cumulative production and production rate are used as the starting point for this projection. The cumulative production variable is increased by increments of 1 percent of the last observed value, while the production rate variable begins at 70 percent of the last observed value and is increased by 10 percent increments until it reaches 150 percent of the last observed value.

These projections are printed in matrix form with the projected production rates printed across the top of the matrix and the projected cumulative production plot points printed along the left margin of the matrix. Projected direct labor requirements can then be read directly from the matrix by matching a given production rate with a given number of cumulative units. The value for direct labor requirements is found at the intersection of the corresponding row and column.

In summary, therefore, the program PRODRATE is a modified version of Smith's FORTRAN IV program. Like Smith's program, PRODRATE converts the input data to logarithms prior to regression. In addition, PRODRATE permits the user to automatically receive or decline either or both of the options for Predictive Ability Tests and Projection and Sensitivity Matrices. For the

analyst who is accustomed to working with the learning curve model, the program PRODRATE quickly shows whether the production rate variable is significant and the effect it has on estimating direct labor requirements. The authors believe the program PRODRATE can be a very useful tool for the government cost and price analyst.

Section 2

This section lists the computer program PRODRATE in its entirety. In addition to the modifications which were thoroughly described in the preceding section, great care was taken to document the modified program. The intent of the documentation (i.e., comment lines) was to make it easier for the user to follow particular parts of the program.

```
1000C*****************************************************************************************************
1010C
1020C     PPPPP  RRRRR  OOOOO  DDD    RRRRR  AAAAA  TTTTT  EEEEE
1030C     P   P  R   R  O   O  D  D   R   R  A   A    T    E
1040C     PPPPP  RRRRR  O   O  D   D  RRRRR  AAAAA    T    EEE
1050C     P      R R    O   O  D  D   R R    A   A    T    E
1060C     P      R  R   OOOOO  DDD    R  R   A   A    T    EEEEE
1070C
1080C*****************************************************************************************************
1090C
1100C                    THE CUMULATIVE PRODUCTION AND PRODUCTION RATE COST MODEL
1110C
1120C     THE ORIGINAL PROGRAMMER IS LT COL LARRY L. SMITH [AFIT/LSGM AV# 785-5096] - JAN 1978
1130C     THIS MODIFIED VERSION WAS PROGRAMMED BY CAPT DAVID Y. STEVENS [ESD/PKG AV# 478-3462] -JUNE 1979
1140C*****************************************************************************************************
1150 5 FORMAT(1H1,//,100("*"),//,1X,40X,"PRODRATE INSTRUCTIONS",//,1X,100("*"),//,
1160&  "      THIS PROGRAM IS DESIGNED TO EVALUATE THE VARIATION IN DIRECT LABOR REQUIREMENTS AS A ",/,
1170&  " FUNCTION OF CUMULATIVE PRODUCTION AND PRODUCTION RATE.  IN ADDITION, THE ANALYST MAY ",/,
1180&  " COMPARE THE RESULTS OBTAINED FROM THE STANDARD LEARNING CURVE WITH THE RESULTS OBTAINED ",/,
1190&  " FROM THE CUMULATIVE PRODUCTION AND PRODUCTION RATE MODEL.  THE COST MODELS USED IN THIS ",/,
1200&  " PROGRAM ARE:",//,
1210&  "      1.  REDUCED MODEL [STANDARD LEARNING CURVE MODEL]",//,
1220&  "              Y = B0 * (X1 ** B1) * (10 ** E)",//,
1230&  "      2.  FULL MODEL [CUMULATIVE PRODUCTION AND PRODUCTION RATE MODEL]",//,
1240&  "              Y = B0 * (X1 ** B1) * (X2 ** B2) * (10 ** E)",//,
1250&  "         WHERE: Y               IS THE DIRECT LABOR REQUIREMENTS",/,
1260&  "                X1              IS THE CUMULATIVE PRODUCTION PLOT POINT",/,
1270&  "                X2              IS THE PRODUCTION RATE PROXY(E.G. EQUIVALENT UNITS PER MONTH)",/,
1280&  "                E               REPRESENTS THE ERROR TERM",/,
1290&  "                B0, B1, AND B2  ARE  PARAMETERS DETERMINED BY REGRESSION",//,
1300&  "      DATA ARE INPUT BY READING FROM ANY PROPERLY FORMATTED DATA FILE.  YOUR DATA FILE SHOULD ",/,
1310&  " BE SAVED TO ANY PERMANENT FILENAME.  YOU WILL BE ASK TO INPUT THE NAME OF YOUR DATA FILE ",/,
1320&  " AT THE APPROPRIATE STEP IN THE PROGRAM.  THE NAME OF YOUR DATA FILE CAN NOT EXCEED 8 ",/,
1330&  " CHARACTERS.  THE FIRST LINE OF THE DATA FILE MUST CONTAIN A LINE NUMBER AND THE NUMBER OF ",/,
1340&  " CASES TO BE READ.  THE DATA IS THEN ENTERED ONE CASE PER LINE IN THE FOLLOWING ORDER: ",/,
1350&  " LINE NUMBER, OBSERVED DIRECT LABOR REQUIREMENT (Y), CUMULATIVE PRODUCTION PLOT POINT (X1),",/,
1360&  " AND PRODUCTION RATE PROXY (X2).  THE PROGRAM USES A FREE FIELD READ FORMAT; THEREFORE,",/,
1370&  " EACH VARIABLE MUST BE SEPARATED BY AT LEAST ONE SPACE (OR OTHER DELIMITER) BUT NO OTHER",/,
1380&  " SPECIAL FORMAT IS REQUIRED.  AN EXAMPLE OF A DATA FILE WITH 5 CASES IS PRESENTED BELOW:",//,
1390&  "      100     5",/,
1400&  "      101     100     9.5     9.5",/,
1410&  "      102      90     30      20.5",/,
1420&  "      103      80     55      25",/,
1430&  "      104      75     82      27",/,
1440&  "      105      71    113      31",//,
1450&  " ONE ADVANTAGE OF THIS PROGRAM IS THAT THE RESULTS OBTAINED WILL BE IN THE SAME UNITS AND",/,
1460&  " FORM AS THE INPUT DATA.  FOR EXAMPLE, IF YOU ARE WORKING IN DIRECT LABOR HOURS PER MONTH",/,
1470&  " AND EQUIVALENT UNITS, THE RESULTS WILL BE IN TERMS OF THESE UNITS.  ALSO, IF YOU WISH TO USE",/,
1480&  " A CUMULATIVE AVERAGE APPROACH, ALL YOU NEED DO IS AGGREGATE THE DATA BASE IN THAT MANNER.",//,
1490&  "      THE PROGRAM BEGINS BY TRANSFORMING THE INPUT DATA TO COMMON LOGARITHMS.  LOG LINEAR",/,
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& " REGRESSION IS THEN PERFORMED AS FOLLOWS:  Y  REGRESSED ON  X1, Y  REGRESSED ON  X2, AND",/,
& " FINALLY Y  REGRESSED ON BOTH X1 AND X2.  OBSERVED DIRECT LABOR REQUIREMENTS, PREDICTED",/,
& " DIRECT LABOR REQUIREMENTS, AND RESIDUALS ARE PRINTED IN ORIGINAL (UNTRANSFORMED) FORM FOR",/,
& " EACH REGRESSION SITUATION.  IN ADDITION, SUMMARY STATISTICS ARE PRINTED FOR EACH MODEL.  THE",/,
& " SUMMARY STATISTICS INCLUDE TWO COEFFICIENTS OF DETERMINATION R SQUARED LOG AND R SQUARED",/,
& " ACTUAL.  THE R SQUARED LOG REPRESENTS THE GOODNESS OF FIT OF THE MODEL TO THE TRANSFORMED",/,
& " DATA (LOG FORM).  THE R SQUARED ACTUAL, ON THE OTHER HAND, IS COMPUTED USING THE",/,
& " UNTRANSFORMED RESIDUALS, AND IS REPRESENTATIVE OF HOW WELL THE MODEL FITS THE UNTRANSFORMED",/,
& " DATA.")
6 FORMAT(1H1,1X,//,
& "      SEVERAL OPTIONS ARE AVAILABLE WITHIN THIS PROGRAM AND CAN BE SELECTED BY APPROPRIATE",/,
& " ANSWERS TO THE FOLLOWING QUESTIONS:",//,
& "      1.  DO YOU WANT TO CHECK DATA AS IT IS READ FROM FILE ........ AND CONVERTED TO",/,
& "      LOGARITHMS?",//,
& "      YES - WILL CAUSE THE PRINTING OF A LISTING OF THE RATIONAL INPUT DATA AND THE",/,
& "            ASSOCIATED LOGARITHMIC VALUES.",//,
& "      NO  - SUPPRESSES THIS OPTION.",//,
& "      2.  DO YOU WANT A COMPARISON OF THE SHORTRANGE PREDICTIVE ABILITY OF THE TWO MODELS?",//,
& "      YES - WILL CAUSE THE PREDICTIVE ABILITY TEST OPTION TO BE ACTIVATED AND THE USER WILL",/,
& "            BE ASKED: 'HOW MANY CASES DO YOU WISH TO TRUNCATE?'  THE RESPONSE TO THIS",/,
& "            QUESTION MAY BE ANY INTEGER VALUE GREATER THAN OR EQUAL TO 2.  THE PREDICTIVE",/,
& "            ABILITY TEST SIMULATES FUTURE PREDICTIONS BY PERFORMING A STEPWISE TRUNCATION OF",/,
& "            THE HISTORICAL DATA.  FOR THIS REASON, AN UPPER LIMITATION ON THE NUMBER OF",/,
& "            CASES TRUNCATED WOULD BE: ((TOTAL NUMBER OF CASES IN DATA FILE) / 2) - 2",/,
& "            FOR EXAMPLE, IF YOUR DATA FILE CONTAINS 50 CASES, YOUR UPPER LIMIT WOULD BE",/,
& "            23 CASES.  THIS, OF COURSE, REPRESENTS ONLY THE MAXIMUM NUMBER OF CASES THAT",/,
& "            COULD BE TRUNCATED.  IN PRACTICE YOU MAY WANT TO TRUNCATE ONLY A SMALL NUMBER OF",/,
& "            CASES.  THUS, IF YOUR DATA IS COLLECTED IN MONTHLY INTERVALS, YOU CAN LOOK AT",/,
& "            THE PREDICTIVE ABILITY OF THE FULL AND REDUCED MODELS FOR AN 18 MONTH TIME SPAN BY",/,
& "            SPECIFYING '18'.  LIKEWISE, IF YOUR DATA IS COLLECTED IN QUARTERS, YOU CAN LOOK",/,
& "            AT THE PREDICTIVE ABILITY OF BOTH MODELS FOR AN 18 MONTH TIME SPAN BY SPECIFYING",/,
& "            '6'.  AFTER ALL PREDICTIVE ABILITY TEST SITUATIONS ARE PRINTED, THE PROGRAM",/,
& "            PRINTS A SUMMARY OF THE TEST RESULTS.",//,
& "      NO  - SUPPRESSES THIS OPTION.",//,
& "      3.  DO YOU WANT PROJECTION AND SENSITIVITY MATRIX?",//,
& "      YES - WILL CAUSE PRINTING OF PROJECTION AND SENSITIVITY MATRIX.  THIS MATRIX PRESENTS",/,
& "            PROJECTED DIRECT LABOR REQUIREMENTS FOR SELECTED PAIRS OF CUMULATIVE PRODUCTION",/,
& "            PLOT POINTS AND PRODUCTION RATES.  THE PROJECTION INTERVAL FOR THE CUMULATIVE",/,
& "            PRODUCTION PLOT POINT IS 1% OF THE LAST OBSERVED VALUE.  THE PROJECTION VALUES",/,
& "            FOR PRODUCTION RATE ARE 70, 80, 90, 100, 110, 120, 130, 140, AND 150 PERCENT OF",/,
& "            THE LAST OBSERVED VALUE OF PRODUCTION RATE.",//,
& "      NO  - SUPPRESSES THIS OPTION.")
C
C
C*****************************************************************************
C     DIMENSIONING VARIABLES
C*****************************************************************************
      ALPHA ANSWER(10)
      FILENAME DATAFILE
      DIMENSION PLOT(99),RATE(99),HRS(99),Y(99),X1(99),X2(99),LN(99),
&     NEWPLOT(100),PRORATE(9),FHRS(100,9),ADEVR(999),ADEVF(999)
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      DATA SUMHRS,SUMX1,SUMX2,SUMY,SSX1,SSX2,SUMX1Y,SUMX2Y,SMX1X2,
     &SSE,SSE1,SSE2,SSEL,SSEL1,SSEL2,SSTO,SSTO1,SSTO2,SSTOL,SSTOL1,SSTOL2/21*0/
C***********************************************************************************************
C
C     PART I - BEGIN PROGRAM, INSTRUCTIONS, DATA INPUT, DATA TRANSFORMATION, AND OPTION SELECTIONS.
C
C***********************************************************************************************
      PRINT,"                         THE CUMULATIVE PRODUCTION AND PRODUCTION RATE COST MODEL"
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     INSTRUCTIONS OPTION SELECTION
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      PRINT," "
      PRINT 10
   10 FORMAT(1X,"DO YOU WANT INSTRUCTIONS?",//)
  100 INPUT, ANSWER(1)
      IF (ANSWER(1).EQ."NO") GO TO 102
      IF (ANSWER(1).EQ."YES") GO TO 101
      PRINT," ANSWER YES OR NO ONLY PLEASE"
      PRINT," "
      GO TO 100
  101 PRINT 5
      PRINT 6
      PRINT 70
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     INPUT THE DATA AND TRANSFORM THE VARIABLES TO LOGARITHMS
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C
  102 PRINT 20
   20 FORMAT(1X,"PLEASE ENTER THE NAME OF YOUR DATA FILE",//)
      INPUT, DATAFILE
      READ(DATAFILE,25)LN(1),NCASES
   25 FORMAT(V)
     DO 30 I=1,NCASES
      READ(DATAFILE,25)LN(I),HRS(I),PLOT(I),RATE(I)
      Y(I)  = ALOG10(HRS(I))
      X1(I) = ALOG10(PLOT(I))
      X2(I) = ALOG10(RATE(I))
      SUMHRS = SUMHRS + HRS(I)
      SUMX1  = SUMX1  + X1(I)
      SUMX2  = SUMX2  + X2(I)
      SUMY   = SUMY   + Y(I)
      SSX1   = SSX1   + X1(I)**2
      SSX2   = SSX2   + X2(I)**2
      SSY    = SSY + Y(I)**2
      SUMX1Y = SUMX1Y + X1(I)*Y(I)
      SUMX2Y = SUMX2Y + X2(I)*Y(I)
      SMX1X2 = SMX1X2 + X1(I)*X2(I)
   30CONTINUE
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     DATA CHECK OPTION SELECTION
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
```

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      PRINT 35,DATAFILE
   35 FORMAT(1X,"DO YOU WANT TO CHECK DATA AS IT IS READ FROM FILE ",A8," AND CONVERTED TO LOGARITHMS?",//)
  103 INPUT,ANSWER(2)
      IF (ANSWER(2).EQ."NO") GO TO 104
      IF (ANSWER(2).EQ."YES") GO TO 104
      PRINT," ANSWER YES OR NO ONLY PLEASE"
      PRINT," "
      GO TO 103
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     PREDICTIVE ABILITY TEST OPTION SELECTION
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  104 PRINT 40
   40 FORMAT(1X,"DO YOU WANT A COMPARISON OF THE SHORTRANGE PREDICTIVE ABILITY OF THE TWO MODELS?",//)
  105 INPUT,ANSWER(3)
      IF (ANSWER(3).EQ."NO") GO TO 106
      IF (ANSWER(3).EQ."YES") GO TO 203
      PRINT," ANSWER YES OR NO ONLY PLEASE"
      PRINT," "
      GO TO 105
  203 PRINT 42
   42 FORMAT(1X,"HOW MANY CASES DO YOU WISH TO TRUNCATE?",//)
      INPUT,ITRUNC
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     PROJECTION AND SENSITIVITY MATRIX OPTION SELECTION
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  106 PRINT 45
   45 FORMAT(1X,"DO YOU WANT PROJECTION AND SENSITIVITY MATRIX?",//)
  107 INPUT,ANSWER(4)
      IF (ANSWER(4).EQ."NO") GO TO 108
      IF (ANSWER(4).EQ."YES") GO TO 108
      PRINT," ANSWER YES OR NO ONLY PLEASE"
      PRINT," "
      GO TO 107
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     BEGIN DATA CHECK OPTION
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
  108 IF (ANSWER(2).EQ."NO") GO TO 109
      PRINT 50, DATAFILE
   50 FORMAT(1H1,//,75("*"),/,5X,"INPUT DATA AS READ FROM FILE ",A8," AND CONVERTED TO LOGARITHMS",
     &/,75("*"))
      PRINT," LINE    DIRECT LABOR HOURS  * CUM PROD PLOT POINT * PRODUCTION RATE"
      PRINT," NUMBER  RATIONAL  LOGARITHM * RATIONAL  LOGARITHM * RATIONAL  LOGARITHM"
      DO 60 I=1,NCASES
      PRINT 55,LN(I),HRS(I),Y(I),PLOT(I),X1(I),RATE(I),X2(I)
   55 FORMAT(1X,1X,I3,5X,F8.2,2X,F9.7," * ",F8.2,2X,F9.7," * ",F8.2,2X,F9.7)
   60 CONTINUE
      PRINT 65
   65 FORMAT (1X,75("*"))
  109 PRINT 70
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   70 FORMAT(1H1,15X)
C*****************************************************************************************
C
C     PART II - PEARSON CORRELATION COEFFICIENTS AND REGRESSION ANALYSIS
C
C*****************************************************************************************
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT PEARSON CORRELATION COEFFICIENTS
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      RX1Y  = (SUMX1Y-SUMX1*SUMY/NCASES)/SQRT((SSX1-(SUMX1**2/NCASES))*(SSY-(SUMY**2/NCASES)))
      RX2Y  = (SUMX2Y-SUMX2*SUMY/NCASES)/SQRT((SSX2-(SUMX2**2/NCASES))*(SSY-(SUMY**2/NCASES)))
      RX1X2 = (SMX1X2-SUMX1*SUMX2/NCASES)/SQRT((SSX1-(SUMX1**2/NCASES))*(SSX2-(SUMX2**2/NCASES)))
      RX1X1 = 1.0
      RX2X2 = 1.0
      RYY   = 1.0
      PRINT 71,RYY,RX1Y,RX2Y,RX1Y,RX1X1,RX1X2,RX2Y,RX1X2,RX2X2
   71 FORMAT(1X,///,1X,45("*"),/,4X,"PEARSON CORRELATION COEFFICIENTS ",
&     "MATRIX",/,1X,45("*"),//,6X,"*",5X,"Y",6X,"*",5X,"X1",5X,"*",5X,
&     "X2",/,1X,45("*"),/,2X,"Y",3X,3("* ",F10.7,1X),/,1X,45("*"),/,2X,
&     "X1",2X,3("* ",F10.7,1X),/,1X,45("*"),/,2X,"X2",2X,3("* ",F10.7,1X),////)
      PRINT 70
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT THE REGRESSION RESULTS OF THE STANDARD LEARNING CURVE MODEL
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      B1 =(SUMX1Y-((SUMX1*SUMY)/NCASES))/(SSX1-(SUMX1**2/NCASES))
      YBAR = SUMY/NCASES
      HRSBAR = SUMHRS/NCASES
      X1BAR = SUMX1/NCASES
      X2BAR = SUMX2/NCASES
      B0 = YBAR- B1*X1BAR
      A00 = 10.**B0
      PRINT 75
   75 FORMAT(1X,75("*"),/,14X,"RESULTS OF THE STANDARD LEARNING",
&     " CURVE MODEL",/,1X,75("*"),/,1X,"CASE",3X,"OBSERVED",5X,"PREDICTED",
&     5X,"RESIDUAL",5X,"% DEVIATION")
      DO 110 I=1,NCASES
      YHATL  = B0 + B1 * X1(I)
      RESIDL = Y(I) - YHATL
      SSEL1  = SSEL1 + RESIDL ** 2
      SSTOL1 = SSTOL1 + (Y(I) - YBAR) ** 2
      YHAT   = 10 ** YHATL
      RESID  = HRS(I) - YHAT
      PERCENT= (RESID / HRS(I)) * 100
      SSE1   = SSE1 + RESID ** 2
      SSTO1  = SSTO1 + (HRS(I) - HRSBAR) ** 2
      PRINT 80,I,HRS(I),YHAT,RESID,PERCENT
   80 FORMAT(1X,I3,4X,F8.2,6X,F8.2,5X,F8.2,7X,F6.2)
  110 CONTINUE
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT STATISTICS FOR THE STANDARD LEARNING CURVE MODEL
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
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      NOFD  = NCASES - 2
      TMSRL = (SSTOL1 - SSEL1)
      TMSEL = SSEL1 / NOFD
      SEE   = SQRT (TMSEL)
      VARB0 = SEE / (1 / NCASES + X1BAR **2 / (SSX1 - (SUMX1 ** 2 / NCASES)))
      SEB0  = SQRT (VARB0)
      SEB1  = SEE / (SQRT(SSX1-(SUMX1**2/NCASES)))
      RSQL1 = (SSTOL1 - SSEL1) / SSTOL1
      RSQA1 = (SSTO1- SSE1) / SSTO1
      FRATIO= TMSRL / TMSEL
      PLEARN= (10 ** (B1 + ALOG10(2.0))) * 100
      PRINT 81,B0,SEB0,AB0,B1,SEB1,RSQL1,SEE,TMSEL,TMSRL,FRATIO,NOFD,RSQA1,PLEARN
   81 FORMAT(1X,75("*"),/,1X,"THE EQUATION FOR THIS MODEL IS: ",
&     "      YHAT = B0 * X1 ** B1",/,1X,
&     "IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B1 * LOG(X1)",
&     /,1X,"WHERE: LOG(B0) =",F8.5,4X,"STD ERROR =",F8.5,4X,"B0 =",F11.5,
&     /,13X,"B1 =",F8.5,4X,"STD ERROR =",F8.5,
&     /,1X,"SUMMARY STATISTICS:",/,1X,
&     "R SQUARED LOG     =",F7.5,10X,"STD ERROR EST =",F11.4,/,1X,
&     "MSE",13X,"=",F9.5,8X,"MSR",11X,"=",F9.5,/,1X,
&     "F RATIO",9X,"=",F9.4,8X,"D. F. (N/D)    = 1/",I3,/1X,
&     "R SQUARED ACTUAL=",F7.5,8X,"LEARNING FACTOR =",F9.5," PERCENT",
&     /,1X,75("*"))
      PRINT 70
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT THE REGRESSION RESULTS FOR THE REDUCED HRS VS RATE MODEL
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      B2 =(SUMX2Y-((SUMX2*SUMY)/NCASES))/(SSX2-(SUMX2**2/NCASES))
      B0 = YBAR - B2 * X2BAR
      AB0= 10 ** B0
      PRINT 82
   82 FORMAT(1X,75("*"),/,11X,"RESULTS OF REGRESSION ON PRODUCTION",
&     " RATE VARIABLE ALONE",/,1X,75("*"),/,1X,"CASE",3X,"OBSERVED",5X,
&     "PREDICTED",5X,"RESIDUAL",5X,"% DEVIATION")
     DO 111 I=1,NCASES
      YHATL  = B0 + B2 * X2(I)
      RESIDL = Y(I) - YHATL
      SSEL2   = SSEL2 + RESIDL **2
      SSTOL2  = SSTOL2 + (Y(I) - YBAR) ** 2
      YHAT   = 10 ** YHATL
      RESID  = HRS(I) - YHAT
      PERCENT= (RESID / HRS(I)) * 100
      SSE2   = SSE2 + RESID **2
      SSTO2  = SSTO2 + (HRS(I) - HRSBAR) ** 2
      PRINT 80,I,HRS(I),YHAT,RESID,PERCENT
  111 CONTINUE
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT STATISTICS FOR THE REDUCED HRS VS RATE MODEL
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
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      THSRL = (SSTOLZ - SSELZ)
      THSEL = SSELZ / NDFB
      SEE   = SQRT(THSEL)
      VARB0 = SEE / (1 / NCASES + X2BAR ** 2 / (SSX2 - (SUMX2 ** 2 / NCASES)))
      SEB0  = SQRT (VARB0)
      SEB2  = SEE / (SQRT (SSX2 - (SUMX2 ** 2 / NCASES)))
      RSQLZ = (SSTOLZ - SSELZ) / SSTOLZ
      RSQAZ = (SSTOZ- SSEZ) / SSTOZ
      FRATIO= THSRL / THSEL
      PRINT 83,B0,SEB0,AB0,B2,SEB2,RSQLZ,SEE,THSEL,THSRL,FRATIO,NDFB,RSQAZ
   83 FORMAT(1X,75("*"),/,1X,"THE EQUATION FOR THIS MODEL IS: ",
&     "     YHAT = B0 * X2 ** B2",/,1X
&     "IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B2 * LOG(X2)",
&     /,1X,"WHERE: LOG(B0) =",F8.5,4X,"STD ERROR =",F8.5,4X,"B0 =",F11.5,
&     /,13X,"B2 =",F8.5,4X,"STD ERROR =",F8.5,
&     /,1X,"SUMMARY STATISTICS:",/,1X,
&     "R SQUARED LOG    =",F7.5,10X,"STD ERROR EST =",F11.4,/,1X,
&     "MSE",13X,"=",F9.5,8X,"MSR",11X,"=",F9.5,/,1X,
&     "F RATIO",9X,"=",F9.4,8X,"D. F. (N/D)    = 1/",I3,/,1X,
&     "R SQUARED ACTUAL=",F7.5,/,1X,75("*"))
      PRINT 70
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT THE REGRESSION RESULTS FOR THE FULL MODEL
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      DENOM = ((SSX1-X1BAR*SUMX1)*(SSX2-X2BAR*SUMX2) - (SMX1X2-X1BAR*SUMX2)**2)
      B1 =((SSX2-X2BAR*SUMX2)*(SUMX1Y-X1BAR*SUMY) -
&         (SMX1X2-X1BAR*SUMX2)*(SUMX2Y-X2BAR*SUMY))/DENOM
      B2 = ((SSX1-X1BAR*SUMX1) *(SUMX2Y-X2BAR*SUMY) -
&        (SMX1X2-X1BAR*SUMX2)*(SUMX1Y-X1BAR*SUMY))/DENOM
      B0 = YBAR-B1*X1BAR-B2*X2BAR
      AB0 = 10.**B0
      PRINT 84
   84 FORMAT(1X,75("*"),/,6X,"RESULTS OF COMBINED CUMULATIVE PRODUCTION",
&     " AND PRODUCTION RATE MODEL",/,1X,75("*"),/,1X,"CASE",3X,"OBSERVED",5X,
&     "PREDICTED",5X,"RESIDUAL",5X,"% DEVIATION")
     DO 112 I=1,NCASES
      THATL = B0 + B1 * X1(I) + B2 * X2(I)
      RESIDL= Y(I) - THATL
      SSEL  = SSEL + RESIDL ** 2
      SSTOL = SSTOL + (Y(I) - YBAR) ** 2
      THAT  = 10 ** THATL
      RESID = HRS(I) - THAT
      PERCENT= (RESID / HRS(I)) * 100
      SSE   = SSE + RESID ** 2
      SSTO  = SSTO + (HRS(I) - HRSBAR) ** 2
      PRINT 80,I,HRS(I),THAT,RESID,PERCENT
  112 CONTINUE
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT STATISTICS FOR THE FULL MODEL
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      NDFB  = NCASES - 3
                                                              .
                                                              .
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      TMSRL = (SSTOL - SSEL) / 2
      TMSEL = SSEL / NDFD
      SEE   = SQRT(TMSEL)
      ZVAL  = NCASES*(SSX1 * SSX2 - SMX1X2 ** 2) - SUMX1*(SUMX1 * SSX2 -
&             SMX1X2 * SUMX2) + SUMX2*(SUMX1 * SMX1X2 - SSX1 * SUMX2)
      AVAL  = (SSX1 * SSX2 - SMX1X2 ** 2) / ZVAL
      VARB0 = TMSEL * AVAL
      SEB0  = SQRT (VARB0)
      SEB1  = SQRT((TMSEL * (SSX2 - X2BAR * SUMX2)) / DENOM)
      SEB2  = SQRT((TMSEL * (SSX1 - X1BAR * SUMX1)) / DENOM)
      RSQL  = (SSTOL - SSEL) / SSTOL
      RSQA  = (SSTO - SSE) / SSTO
      FRATIO= TMSRL /TMSEL
      FB1   = (RSQL - RSQL2) / ((1 - RSQL) / (NCASES - 3))
      FB2   = (RSQL - RSQL1) / ((1 - RSQL) / (NCASES - 3))
      PRINT 85,B0,SEB0,AB0,B1,SEB1,FB1,B2,SEB2,FB2,RSQL,SEE,TMSEL,TMSRL,FRATIO,NDFD,RSQA
   85 FORMAT(1X,75("*"),/,1X,"THE EQUATION FOR THIS MODEL IS: ",
&     "      YHAT = B0 * X1 ** B1 * X2 **B2",/,1X,
&     "IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B1 * LOG(X1) + B2 * LOG(X2)",
&     /,1X,"WHERE: LOG(B0) =",F8.5,4X,"STD ERROR =",F8.5,4X,"B0 =",F11.5,
&     /,13X,"B1 =",F8.5,4X,"STD ERROR =",F8.5,4X,"F* =",F10.4,/,
&     13X,"B2 =",F8.5,4X,"STD ERROR =",F8.5,4X,"F* =",F10.4,/,1X,
&     "SUMMARY STATISTICS:",/,1X,"R SQUARED LOG    =",F7.5,10X,
&     "STD ERROR EST =",F11.4,/,1X,"MSE",13X,"=",F9.5,8X,"MSR",11X,"=",F9.5,/,1X,
&     "F RATIO",9X,"=",F9.4,8X,"D. F. (N/D)    = 2/",I3,/,1X,
&     "R SQUARED ACTUAL=",F7.5,/,1X,75("*"))
      PRINT 70
C***************************************************************************************************
C
C     PART III - PREDICTIVE ABILITY TEST OPTION
C
C***************************************************************************************************
      IF (ANSWER(3).EQ."NO") GO TO 116
      DO 113 I=1,ITRUNC
      ITEST = NCASES + 1 - I
      PRINT 86,ITEST,HRS(ITEST)
   86 FORMAT(1X,116("*"),/,1X,"*",37X,"SHORTRANGE PREDICTIVE ABILITY ",
&     "COMPARISON",37X,"*",/,1X,"*",16X,
&     "THE DATA PRESENTED BELOW IS FOR CASE #",I3," WHICH HAS AN OBSERVED",
&     " VALUE OF:",F9.2,16X,"*",/,
&     1X,116("*"),/1X,"*",3X,"#",3X,"*",9X,"REDUCED (LEARNING CURVE) ",
&     "MODEL",8X,"**",3X,"FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) ",
&     "MODEL",2X,"*",/,1X,"*"," CASES ",108("*"),/,1X,"*  USED * ",
&     "PREDICTION * % DEVIATION *  EST B0  * EST B1  ** ",
&     "PREDICTION * % DEVIATION *  EST B0  * EST B1  * EST B2  *",
&     /,1X,116("*"))
      DO 114 J=1,ITRUNC
      ICASES = ITEST - J
      SUMY   = 0
```

```
      SUMX1  = 0
      SUMX2  = 0
      SSX1   = 0
      SSX2   = 0
      SUMX1Y = 0
      SUMX2Y = 0
      SMX1X2 = 0
      DO 115 K=1,ICASES
      SUMY   = SUMY + Y(K)
      SUMX1  = SUMX1 + X1(K)
      SUMX2  = SUMX2 + X2(K)
      SSX1   = SSX1 + X1(K) ** 2
      SSX2   = SSX2 + X2(K) ** 2
      SUMX1Y = SUMX1Y + X1(K) * Y(K)
      SUMX2Y = SUMX2Y + X2(K) * Y(K)
      SMX1X2 = SMX1X2 + X1(K) * X2(K)
  115 CONTINUE
      ICOUNTA = ICOUNTA + 1
      YBAR  = SUMY / ICASES
      X1BAR = SUMX1 / ICASES
      X2BAR = SUMX2 / ICASES
      B1R   = (SUMX1Y - ((SUMX1 * SUMY) / ICASES)) / (SSX1 - (SUMX1 ** 2 / ICASES))
      B0R   = YBAR - B1R * X1BAR
      AB0R  = 10 ** B0R
      THATR = 10 ** (B0R + B1R * X1(ITEST))
      DEVR    = HRS(ITEST) - THATR
      ADEVR(ICOUNTA) = ABS(DEVR)
      SUMADEVR = SUMADEVR + ADEVR(ICOUNTA)
      PDEVR    = 100 * DEVR/HRS(ITEST)
      APDEVR = ABS(PDEVR)
      IF (APDEVR.GT.10.0) GO TO 201
      ICOUNTCR = ICOUNTCR + 1
      IF (APDEVR.GT.5.0) GO TO 201
      ICOUNTER = ICOUNTER + 1
  201 DENOM = ((SSX1-X1BAR*SUMX1)*(SSX2-X2BAR*SUMX2) - (SMX1X2-X1BAR*SUMX2)**2)
      B1F   = ((SSX2-X2BAR*SUMX2)*(SUMX1Y-X1BAR*SUMY) -
     &            (SMX1X2-X1BAR*SUMX2)*(SUMX2Y-X2BAR*SUMY))/DENOM
      B2F   = ((SSX1-X1BAR*SUMX1)*(SUMX2Y-X2BAR*SUMY) -
     &            (SMX1X2-X1BAR*SUMX2)*(SUMX1Y-X1BAR*SUMY))/DENOM
      B0F   = YBAR - B1F * X1BAR - B2F * X2BAR
      AB0F  = 10 ** B0F
      THATF = 10  ** (B0F + B1F * X1(ITEST) + B2F * X2(ITEST))
      DEVF  = HRS(ITEST) - THATF
      ADEVF(ICOUNTA) = ABS(DEVF)
      SUMADEVF = SUMADEVF + ADEVF(ICOUNTA)
      PDEVF = 100 * DEVF/HRS(ITEST)
      APDEVF = ABS(PDEVF)
      IF (APDEVF.GT.10.0) GO TO 202
      ICOUNTGF = ICOUNTGF + 1
      IF (APDEVF.GT.5.0) GO TO 202
      ICOUNTEF = ICOUNTEF + 1
```

```
  202 PRINT 87,ICASES,THATR,PDEVR,AB0R,B1R,THATF,PDEVF,AB0F,B1F,B2F
   87 FORMAT(1X,"*",2X,I3,2X,"*",1X,F9.2,2X,"*",3X,F6.2,4X,"*",F9.2,1X,
&     "*",F8.5,1X,"**",1X,F9.2,2X,"*",3X,F6.2,4X,"*",F9.2,1X,"*",F8.5,1X,
&     "*",F8.5,1X,"*")
  114 CONTINUE
      PRINT 88
   88 FORMAT(1X,116("*"),//////)
      COUNT = COUNT + 1.0
      FLAG1 = COUNT / 2.0
      FLAG2 = FLAG1 - INT(FLAG1)
      IF (FLAG2.NE.0.0) GO TO 113
      PRINT 70
  113 CONTINUE
      AVGADEVR = SUMADEVR / ICOUNTA
      AVGADEVF = SUMADEVF / ICOUNTA
     DO 119 I =1,ICOUNTA
      SSDEVR  = SSDEVR + (ADEVR(I) - AVGADEVR)**2
      SSDEVF  = SSDEVF + (ADEVF(I) - AVGADEVF)**2
  119 CONTINUE
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C     CALCULATE AND PRINT RESULTS SUMMARY FOR PREDICTIVE ABILITY TESTS
C::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      VARADEVR = SSDEVR / (ICOUNTA - 1)
      VARADEVF = SSDEVF / (ICOUNTA - 1)
      TESTSTAT = (AVGADEVR-AVGADEVF)/SQRT((VARADEVR/ICOUNTA)+(VARADEVF/ICOUNTA))
      PCENTER = 100 * ICOUNTER / ICOUNTA
      PCENTGR = 100 * ICOUNTGR / ICOUNTA
      PCENTEF = 100 * ICOUNTEF / ICOUNTA
      PCENTGF = 100 * ICOUNTGF / ICOUNTA
      PRINT 95,AVGADEVR,AVGADEVF,VARADEVR,VARADEVF,TESTSTAT,ICOUNTA,
&     ICOUNTA,ICOUNTER,ICOUNTEF,PCENTER,PCENTEF,ICOUNTGR,ICOUNTGF,PCENTGR,
&     PCENTGF
   95 FORMAT(1X,67("*"),/,1X,"*",10X,"SUMMARY OF PREDICTIVE ABILITY TESTS",
&     " RESULTS",12X,"*",/,1X,67("*"),/,1X,"*",9X,"ITEMS OF INTEREST",8X,
&     "* REDUCED MODEL *  FULL MODEL  *",/,1X,67("*"),/,1X,"* AVERAGE ",
&     "ABSOLUTE DEVIATION",7X,"*",3X,F9.2,3X,"*",2X,F9.2,3X,"*",/,1X,
&     "* VARIANCE OF ABSOLUTE DEVIATIONS",2X,"*",1X,F11.2,3X,"*",F11.2,3X,
&     "*",/,1X,"* TEST STATISTIC (SEE NOTE)",8X,"*",6X,"---",6X,"*",2X,
&     F9.2,3X,"*",/,1X,"* TOTAL NUMBER OF TEST SITUATIONS  *",6X,I3,6X,"*",
&     5X,I3,6X,"*",/,1X,"* NUMBER OF PREDICTIONS WITHIN 5%  *",6X,I3,6X,
&     "*",5X,I3,6X,"*",/,1X,"* PERCENT OF PREDICTIONS WITHIN 5% *",6X,F4.0,
&     5X,"*",5X,F4.0,5X,"*",/,1X,"* NUMBER OF PREDICTIONS WITHIN 10% *",
&     6X,I3,6X,"*",5X,I3,6X,"*",/,1X,"* PERCENT OF PREDICTIONS WITHIN 10%*",
&     6X,F4.0,5X,"*",5X,F4.0,5X,"*",/,1X,67("*"),/,1X,"NOTE: IN TESTING FOR ",
&     "STATISTICAL SIGNIFICANCE USE STUDENT'S T DISTRIBUTION",/,1X
&     "IF THE NUMBER OF TEST SITUATIONS ARE LESS THAN 60; OTHERWISE ",
&     "USE STANDARD",/,1X,"NORMAL DISTRIBUTION.  IN EITHER CASE THIS IS ",
&     "A ONE TAILED TEST.  IF",/,1X,"THE TEST STATISTIC IS GREATER THAN ",
&     "THE CRITICAL STATISTIC ONE MAY",/,1X,"CONCLUDE THAT THE AVERAGE ",
```

```
      "ABSOLUTE DEVIATION OBTAINED WITH THE FULL",/,1X,"MODEL IS ",
      "SIGNIFICANTLY LESS THAN THAT OBTAINED WITH THE REDUCED MODEL.")
C***************************************************************************************
C
C     PART IV - PROJECTION AND SENSITIVITY MATRIX OPTION
C
C***************************************************************************************
  116 IF (ANSWER(4).EQ."NO") GO TO 125
      ADDPLOT = PLOT(NCASES)
      PRINT 70
      DO 117 I=1,100
      ADDPLOT = ADDPLOT + .01 * PLOT(NCASES)
      NEWPLOT(I) = INT(ADDPLOT)
      ADDRATE = .60 * RATE(NCASES)
      DO 118 J=1,9
      ADDRATE = ADDRATE + .1 * RATE(NCASES)
      PRORATE(J) = ADDRATE
      FHRS(I,J)  = B0 * NEWPLOT(I)**B1 * PRORATE(J)**B2
  118 CONTINUE
  117 CONTINUE
      ISTART = 1
      ISTOP  = 50
      DO 120 K=1,2
      PRINT 89,(PRORATE(J),J=1,9)
   89 FORMAT(1X,113("*"),/,1X,"*",39X,"PROJECTION AND SENSITIVITY MATRIX",
      39X,"*",/,1X,113("*"),/,1X,"*  PROJECTED *",36X,"PROJECTED PRODUCTION",
      " RATES",36X,"*",/,1X,"* CUMULATIVE *",99("*"),/,1X,"*    UNITS    *",
      9(F8.2,2X,"*"),/,1X,113("*"))
      DO 121 I=ISTART,ISTOP
      PRINT 90,NEWPLOT(I),(FHRS(I,J),J=1,9)
   90 FORMAT(1X,"*",3X,I6,3X,"*",9(1X,F8.1,1X,"*"))
  121 CONTINUE
      PRINT 91
   91 FORMAT(1X,113("*"))
      PRINT92
   92 FORMAT(1X,"NOTE: 1.  PROJECTED VALUES FOR DIRECT LABOR HOURS MAY ",
      "BE READ FROM THE ABOVE MATRIX BY MATCHING A GIVEN PRODUCTION",/,1X,
      "RATE WITH A GIVEN NUMBER OF CUMULATIVE UNITS AND READING THE ",
      "VALUE FOR DIRECT LABOR HOURS FOUND AT THE INTERSECTION",/,1X,
      "OF THE CORESPONDING ROW AND COLUMN.  FORECASTING MODEL IS THE ",
      "CUMULATIVE PRODUCTION & PRODUCTION RATE MODEL.",/,7X,"2.  PROJECT",
      "ION INTERVAL FOR CUMULATIVE UNITS IS 1% OF THE LAST OBSERVED VALUE",
      " OF CUMULATIVE UNITS.",/,7X,"3.  PROJECTION VALUES FOR PRODUCTION ",
      "RATE ARE 70, 80, 90, 100, 110, 120, 130, 140, AND 150 PERCENT OF ",
      "THE",/,1X,"LAST OBSERVED VALUE OF PRODUCTION RATE.")
      ISTART = 51
      ISTOP  = 100
      PRINT 70
  120 CONTINUE
  125 STOP
      END
```

Section 3

This section illustrates the use of PRODRATE with an artificial data base. The artificial data base was designed to meet two objectives. First, it was designed to approximate an avionics production program. Second, the data base was designed intentionally to make the cumulative production and production rate model clearly superior.

****

PRODRATE INSTRUCTIONS

****

THIS PROGRAM IS DESIGNED TO EVALUATE THE VARIATION IN DIRECT LABOR REQUIREMENTS AS A FUNCTION OF CUMULATIVE PRODUCTION AND PRODUCTION RATE. IN ADDITION, THE ANALYST MAY COMPARE THE RESULTS OBTAINED FROM THE STANDARD LEARNING CURVE WITH THE RESULTS OBTAINED FROM THE CUMULATIVE PRODUCTION AND PRODUCTION RATE MODEL. THE COST MODELS USED IN THIS PROGRAM ARE:

1. REDUCED MODEL [STANDARD LEARNING CURVE MODEL]

   Y = B0 * (X1 ** B1) * (10 ** E)

2. FULL MODEL [CUMULATIVE PRODUCTION AND PRODUCTION RATE MODEL]

   Y = B0 * (X1 ** B1) * (X2 ** B2) * (10 ** E)

WHERE: Y	IS THE DIRECT LABOR REQUIREMENTS
X1	IS THE CUMULATIVE PRODUCTION PLOT POINT
X2	IS THE PRODUCTION RATE PROXY(E.G. EQUIVALENT UNITS PER MONTH)
E	REPRESENTS THE ERROR TERM
B0, B1, AND B2	ARE PARAMETERS DETERMINED BY REGRESSION

DATA ARE INPUT BY READING FROM ANY PROPERLY FORMATTED DATA FILE. YOUR DATA FILE SHOULD BE SAVED TO ANY PERMANENT FILENAME. YOU WILL BE ASK TO INPUT THE NAME OF YOUR DATA FILE AT THE APPROPRIATE STEP IN THE PROGRAM. THE NAME OF YOUR DATA FILE CAN NOT EXCEED 8 CHARACTERS. THE FIRST LINE OF THE DATA FILE MUST CONTAIN A LINE NUMBER AND THE NUMBER OF CASES TO BE READ. THE DATA IS THEN ENTERED ONE CASE PER LINE IN THE FOLLOWING ORDER: LINE NUMBER, OBSERVED DIRECT LABOR REQUIREMENT (Y), CUMULATIVE PRODUCTION PLOT POINT (X1), AND PRODUCTION RATE PROXY (X2). THE PROGRAM USES A FREE FIELD READ FORMAT; THEREFORE, EACH VARIABLE MUST BE SEPARATED BY AT LEAST ONE SPACE (OR OTHER DELIMITER) BUT NO OTHER SPECIAL FORMAT IS REQUIRED. AN EXAMPLE OF A DATA FILE WITH 5 CASES IS PRESENTED BELOW:

```
100   5
101  100    9.5    9.5
102   90   30     20.5
103   80   55     25
104   75   82     27
105   71  113     31
```

ONE ADVANTAGE OF THIS PROGRAM IS THAT THE RESULTS OBTAINED WILL BE IN THE SAME UNITS AND FORM AS THE INPUT DATA. FOR EXAMPLE, IF YOU ARE WORKING IN DIRECT LABOR HOURS PER MONTH AND EQUIVALENT UNITS, THE RESULTS WILL BE IN TERMS OF THESE UNITS. ALSO, IF YOU WISH TO USE A CUMULATIVE AVERAGE APPROACH, ALL YOU NEED DO IS AGGREGATE THE DATA BASE IN THAT MANNER.

THE PROGRAM BEGINS BY TRANSFORMING THE INPUT DATA TO COMMON LOGARITHMS. LOG LINEAR REGRESSION IS THEN PERFORMED AS FOLLOWS: Y REGRESSED ON X1, Y REGRESSED ON X2, AND FINALLY Y REGRESSED ON BOTH X1 AND X2. OBSERVED DIRECT LABOR REQUIREMENTS, PREDICTED DIRECT LABOR REQUIREMENTS, AND RESIDUALS ARE PRINTED IN ORIGINAL (UNTRANSFORMED) FORM FOR EACH REGRESSION SITUATION. IN ADDITION, SUMMARY STATISTICS ARE PRINTED FOR EACH MODEL. THE SUMMARY STATISTICS INCLUDE TWO COEFFICIENTS OF DETERMINATION R SQUARED LOG AND R SQUARED ACTUAL. THE R SQUARED LOG REPRESENTS THE GOODNESS OF FIT OF THE MODEL TO THE TRANSFORMED DATA (LOG FORM). THE R SQUARED ACTUAL, ON THE OTHER HAND, IS COMPUTED USING THE UNTRANSFORMED RESIDUALS, AND IS REPRESENTATIVE OF HOW WELL THE MODEL FITS THE UNTRANSFORMED DATA.

SEVERAL OPTIONS ARE AVAILABLE WITHIN THIS PROGRAM AND CAN BE SELECTED BY APPROPRIATE ANSWERS TO THE FOLLOWING QUESTIONS:

1. DO YOU WANT TO CHECK DATA AS IT IS READ FROM FILE .... AND CONVERTED TO LOGARITHMS?

YES - WILL CAUSE THE PRINTING OF A LISTING OF THE RATIONAL INPUT DATA AND THE ASSOCIATED LOGARITHMIC VALUES.

NO - SUPPRESSES THIS OPTION.

2. DO YOU WANT A COMPARISON OF THE SHORTRANGE PREDICTIVE ABILITY OF THE TWO MODELS?

YES - WILL CAUSE THE PREDICTIVE ABILITY TEST OPTION TO BE ACTIVATED AND THE USER WILL BE ASKED: 'HOW MANY CASES DO YOU WISH TO TRUNCATE?' THE RESPONSE TO THIS QUESTION MAY BE ANY INTEGER VALUE GREATER THAN OR EQUAL TO 2. THE PREDICTIVE ABILITY TEST SIMULATES FUTURE PREDICTIONS BY PERFORMING A STEPWISE TRUNCATION OF THE HISTORICAL DATA. FOR THIS REASON, AN UPPER LIMITATION ON THE NUMBER OF CASES TRUNCATED WOULD BE: ((TOTAL NUMBER OF CASES IN DATA FILE) / 2) - 2 FOR EXAMPLE, IF YOUR DATA FILE CONTAINS 50 CASES, YOUR UPPER LIMIT WOULD BE 23 CASES. THIS, OF COURSE, REPRESENTS ONLY THE MAXIMUM NUMBER OF CASES THAT COULD BE TRUNCATED. IN PRACTICE YOU MAY WANT TO TRUNCATE ONLY A SMALL NUMBER OF CASES. THUS, IF YOUR DATA IS COLLECTED IN MONTHLY INTERVALS, YOU CAN LOOK AT THE PREDICTIVE ABILITY OF THE FULL AND REDUCED MODELS FOR AN 18 MONTH TIME SPAN BY SPECIFYING '18'. LIKEWISE, IF YOUR DATA IS COLLECTED IN QUARTERS, YOU CAN LOOK AT THE PREDICTIVE ABILITY OF BOTH MODELS FOR AN 18 MONTH TIME SPAN BY SPECIFYING '6'. AFTER ALL PREDICTIVE ABILITY TEST SITUATIONS ARE PRINTED, THE PROGRAM PRINTS A SUMMARY OF THE TEST RESULTS.

NO - SUPPRESSES THIS OPTION.

3. DO YOU WANT PROJECTION AND SENSITIVITY MATRIX?

YES - WILL CAUSE PRINTING OF PROJECTION AND SENSITIVITY MATRIX. THIS MATRIX PRESENTS PROJECTED DIRECT LABOR REQUIREMENTS FOR SELECTED PAIRS OF CUMULATIVE PRODUCTION PLOT POINTS AND PRODUCTION RATES. THE PROJECTION INTERVAL FOR THE CUMULATIVE PRODUCTION PLOT POINT IS 1% OF THE LAST OBSERVED VALUE. THE PROJECTION VALUES FOR PRODUCTION RATE ARE 70, 80, 90, 100, 110, 120, 130, 140, AND 150 PERCENT OF THE LAST OBSERVED VALUE OF PRODUCTION RATE.

NO - SUPPRESSES THIS OPTION.

INPUT DATA AS READ FROM FILE TESTDATA AND CONVERTED TO LOGARITHMS

LINE	DIRECT LABOR HOURS		COST PROD PLOT POINT		PRODUCTION RATE	
NUMBER	RATIONAL	LOGARITHM	RATIONAL	LOGARITHM	RATIONAL	LOGARITHM
101	1088.00	3.0366289	50.00	1.6989700	2.27	0.3560259
102	803.00	2.9047155	175.00	2.2430381	3.05	0.5054487
103	641.00	2.8068580	313.00	2.4955443	4.45	0.6483600
104	560.00	2.7481880	454.00	2.6570559	4.94	0.6937269
105	493.00	2.6928469	626.00	2.7965743	5.33	0.7267272
106	462.00	2.6646420	795.00	2.9003671	5.85	0.7671559
107	437.00	2.6404814	1005.00	3.0021661	6.47	0.8109043
108	404.00	2.6063814	1286.00	3.0013473	6.71	0.8267225
109	368.00	2.5658478	1495.00	3.1746412	7.00	0.8450980
110	347.00	2.5403295	1775.00	3.2491983	7.37	0.8674675
111	330.00	2.5185139	2092.00	3.3205617	7.79	0.8915375
112	320.00	2.5051500	2421.00	3.3839948	8.33	0.9206450
113	317.00	2.5010593	2769.00	3.4423229	9.09	0.9585639
114	313.00	2.4955443	3176.00	3.5018605	9.86	0.9938769
115	309.00	2.4899585	3557.00	3.5510839	10.51	1.0216027
116	304.00	2.4828736	3976.00	3.5994464	11.17	1.0480532
117	298.00	2.4742163	4452.00	3.6485552	11.80	1.0718820
118	290.00	2.4623980	4964.00	3.6958318	12.37	1.0923697
119	284.00	2.4533183	5450.00	3.7370335	12.87	1.1095785
120	278.00	2.4440448	5939.00	3.7731734	13.38	1.1264561
121	270.00	2.4313638	6461.00	3.8102997	13.65	1.1351327
122	263.00	2.4199557	6972.00	3.8433574	13.98	1.1455072
123	256.00	2.4082400	7491.00	3.8745398	14.28	1.1547232
124	250.00	2.3979400	8080.00	3.9074114	14.65	1.1658376
125	245.00	2.3891661	8658.00	3.9370161	14.98	1.1755118
126	239.00	2.3783979	9248.00	3.9660478	15.29	1.1844075
127	235.00	2.3710679	9848.00	3.9933480	15.65	1.1945143
128	232.00	2.3654880	10450.00	4.0191163	16.04	1.2052044
129	228.00	2.3579348	11031.00	4.0426149	16.35	1.2135178
130	224.00	2.3502480	11625.00	4.0653938	16.66	1.2216750
131	221.00	2.3443923	12227.00	4.0873199	16.97	1.2296818
132	218.00	2.3384565	12838.00	4.1084974	17.27	1.2372923
133	216.00	2.3344537	13349.00	4.1254487	17.56	1.2445245
134	214.00	2.3304138	13849.00	4.1414184	17.81	1.2506639
135	211.00	2.3242824	14337.00	4.1564583	18.01	1.2555137
136	209.00	2.3201463	14866.00	4.1721941	18.22	1.2605484
137	206.00	2.3138672	15454.00	4.1890489	18.41	1.2650538
138	203.00	2.3074960	16048.00	4.2054289	18.58	1.2690457
139	200.00	2.3010300	16654.00	4.2215196	18.78	1.2736954
140	198.00	2.2966652	17172.00	4.2348298	18.94	1.2773800

PEARSON CORRELATION COEFFICIENTS MATRIX

	Y	X1	X2
Y	1.0000000	-0.9955704	-0.9841305
X1	-0.9955704	1.0000000	0.9964585
X2	-0.9841305	0.9964585	1.0000000

```
**********************************************************************
            RESULTS OF THE STANDARD LEARNING CURVE MODEL
**********************************************************************
```

CASE	OBSERVED	PREDICTED	RESIDUAL	% DEVIATION
1	1088.00	1036.45	51.55	4.74
2	803.00	727.41	75.59	9.41
3	641.00	617.19	23.81	3.71
4	560.00	555.61	4.39	0.78
5	493.00	507.39	-14.39	-2.92
6	462.00	474.25	-12.25	-2.65
7	437.00	443.85	-6.85	-1.57
8	404.00	421.56	-17.56	-4.35
9	368.00	396.72	-28.72	-7.81
10	347.00	377.93	-30.93	-8.91
11	330.00	360.78	-30.78	-9.33
12	320.00	346.19	-26.19	-8.19
13	317.00	333.30	-16.30	-5.14
14	313.00	320.63	-7.63	-2.44
15	309.00	310.52	-1.52	-0.49
16	304.00	300.90	3.10	1.02
17	298.00	291.44	6.56	2.20
18	290.00	282.61	7.39	2.55
19	284.00	275.13	8.87	3.12
20	278.00	268.39	9.61	3.46
21	270.00	262.32	7.68	2.84
22	263.00	256.74	6.26	2.38
23	256.00	251.58	4.42	1.73
24	250.00	246.26	3.74	1.50
25	245.00	241.56	3.44	1.40
26	239.00	237.04	1.96	0.82
27	235.00	232.86	2.14	0.91
28	232.00	228.99	3.01	1.30
29	228.00	225.52	2.48	1.09
30	224.00	222.20	1.80	0.80
31	221.00	219.05	1.95	0.88
32	218.00	216.05	1.95	0.89
33	216.00	213.68	2.32	1.07
34	214.00	211.47	2.53	1.18
35	211.00	209.41	1.59	0.75
36	209.00	207.28	1.72	0.82
37	206.00	205.02	0.98	0.48
38	203.00	202.85	0.15	0.08
39	200.00	200.73	-0.73	-0.37
40	198.00	199.00	-1.00	-0.51

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**********************************************************************
THE EQUATION FOR THIS MODEL IS:      YHAT = B0 * X1 ** B1
IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B1 * LOG(X1)
WHERE: LOG(B0) = 3.49572    STD ERROR = 0.13455    B0 = 3131.25031
           B1 =-0.28262    STD ERROR = 0.00433
SUMMARY STATISTICS:
R SQUARED LOG   =0.99115          STD ERROR EST =     0.0164
MSE             =  0.00027        MSR           =  1.14390
F RATIO         =4255.5115        D. F. (N/D)   = 1/ 38
R SQUARED ACTUAL=0.98981        LEARNING FACTOR = 82.20944 PERCENT
**********************************************************************
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RESULTS OF REGRESSION ON PRODUCTION RATE VARIABLE ALONE

CASE	OBSERVED	PREDICTED	RESIDUAL	% DEVIATION
1	1088.00	974.20	113.29	10.40
2	803.00	658.03	144.97	18.05
3	641.00	590.83	50.17	7.83
4	560.00	546.66	13.34	2.38
5	493.00	516.61	-23.61	-4.79
6	462.00	482.05	-20.05	-4.34
7	437.00	447.25	-10.25	-2.35
8	404.00	435.30	-31.30	-7.75
9	368.00	421.81	-53.81	-14.62
10	347.00	405.95	-58.95	-16.99
11	330.00	389.56	-59.56	-18.05
12	320.00	370.61	-50.61	-15.82
13	317.00	347.31	-30.31	-9.56
14	313.00	326.92	-13.92	-4.45
15	309.00	311.76	-2.76	-0.89
16	304.00	297.95	6.05	1.99
17	298.00	286.04	11.96	4.01
18	290.00	276.17	13.83	4.77
19	284.00	268.15	15.85	5.58
20	278.00	260.51	17.49	6.29
21	270.00	256.67	13.33	4.94
22	263.00	252.15	10.85	4.13
23	256.00	248.20	7.80	3.05
24	250.00	243.52	6.48	2.59
25	245.00	239.52	5.48	2.24
26	239.00	235.90	3.10	1.30
27	235.00	231.85	3.15	1.34
28	232.00	227.64	4.36	1.88
29	228.00	224.42	3.58	1.57
30	224.00	221.31	2.69	1.20
31	221.00	218.29	2.71	1.22
32	218.00	215.47	2.53	1.16
33	216.00	212.81	3.19	1.47
34	214.00	210.59	3.41	1.59
35	211.00	208.85	2.15	1.02
36	209.00	207.05	1.95	0.93
37	206.00	205.46	0.54	0.26
38	203.00	204.06	-1.06	-0.52
39	200.00	202.44	-2.44	-1.22
40	198.00	201.17	-3.17	-1.63

THE EQUATION FOR THIS MODEL IS: THAT = B0 * X2 ** B2

IN LOG FORM THIS MODEL BECOMES: LOG(THAT) = LOG(B0) + B2 * LOG(X2)

WHERE: LOG(B0) = 3.25374 STD ERROR = 0.23963 B0 = 1793.67776

B2 =-0.74385 STD ERROR = 0.02176

SUMMARY STATISTICS:

R SQUARED LOG	=0.96851	STD ERROR EST =	0.0309
MSE	= 0.00096	MSR	= 1.11778
F RATIO	=1168.8245	D. F. (N/D)	= 1/ 38
R SQUARED ACTUAL	=0.95677		

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RESULTS OF COMBINED CUMULATIVE PRODUCTION AND PRODUCTION RATE MODEL
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CASE	OBSERVED	PREDICTED	RESIDUAL	% DEVIATION
1	1088.00	1088.11	-0.11	-0.01
2	803.00	803.06	-0.06	-0.01
3	641.00	640.69	0.31	0.05
4	560.00	560.07	-0.07	-0.01
5	493.00	492.67	0.33	0.07
6	462.00	461.92	0.08	0.02
7	437.00	437.09	-0.09	-0.02
8	404.00	404.12	-0.12	-0.03
9	368.00	368.25	-0.25	-0.07
10	347.00	347.05	-0.05	-0.01
11	330.00	329.60	0.40	0.12
12	320.00	319.60	0.40	0.13
13	317.00	317.49	-0.49	-0.16
14	313.00	313.27	-0.27	-0.09
15	309.00	308.95	0.05	0.02
16	304.00	304.29	-0.29	-0.10
17	298.00	297.86	0.14	0.05
18	290.00	290.42	-0.42	-0.14
19	284.00	283.72	0.28	0.10
20	278.00	278.17	-0.17	-0.06
21	270.00	269.53	0.47	0.17
22	263.00	262.77	0.23	0.09
23	256.00	256.26	-0.26	-0.10
24	250.00	250.26	-0.26	-0.10
25	245.00	244.81	0.19	0.08
26	239.00	239.31	-0.31	-0.13
27	235.00	235.05	-0.05	-0.02
28	232.00	231.61	0.39	0.17
29	228.00	227.88	0.12	0.05
30	224.00	224.37	-0.37	-0.16
31	221.00	221.10	-0.10	-0.05
32	218.00	217.94	0.06	0.03
33	216.00	215.92	0.08	0.04
34	214.00	213.76	0.24	0.11
35	211.00	211.36	-0.36	-0.17
36	209.00	208.85	0.15	0.07
37	206.00	205.85	0.15	0.07
38	203.00	202.83	0.17	0.09
39	200.00	200.18	-0.18	-0.09
40	198.00	197.95	0.05	0.03

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THE EQUATION FOR THIS MODEL IS:      THAT = B0 * X1 ** B1 * X2 **B2
IN LOG FORM THIS MODEL BECOMES: LOG(THAT) = LOG(B0) + B1 * LOG(X1) + B2 * LOG(X2)
WHERE: LOG(B0) = 3.75357    STD ERROR = 0.00112    B0 = 5669.84894
            B1 =-0.59925    STD ERROR = 0.00129    F* =4461.1348
            B2 = 0.84602    STD ERROR = 0.00344    F* =9254.3730
SUMMARY STATISTICS:
R SQUARED LOG   =0.99999          STD ERROR EST =     0.0004
MSE             =  0.00000        MSR           =  0.57705
F RATIO         =8883.1562        D. F. (N/D)   = 2/ 37
R SQUARED ACTUAL=1.00000
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SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 40 WHICH HAS AN OBSERVED VALUE OF: 198.00

# CASES	REDUCED (LEARNING CURVE) MODEL				FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL				
USED	PREDICTION	% DEVIATION	EST B0	EST B1	PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
39	199.06	-0.53	3129.01	-0.28252	197.75	0.03	5670.25	-0.59927	0.84607
38	199.10	-0.56	3127.28	-0.28244	197.96	0.02	5668.40	-0.59916	0.84582
37	199.10	-0.56	3127.32	-0.28244	197.95	0.03	5668.67	-0.59913	0.84569
36	199.05	-0.53	3129.40	-0.28254	197.94	0.03	5669.13	-0.59913	0.84565
35	198.94	-0.47	3133.44	-0.28273	197.93	0.04	5669.31	-0.59918	0.84552
34	198.83	-0.42	3137.52	-0.28292	197.95	0.02	5668.95	-0.59917	0.84581
33	198.64	-0.32	3144.40	-0.28324	197.93	0.03	5669.46	-0.59914	0.84566
32	198.44	-0.22	3151.45	-0.28357	197.93	0.04	5669.40	-0.59910	0.84553
31	198.25	-0.13	3158.26	-0.28389	197.92	0.04	5669.41	-0.59908	0.84543
30	198.04	-0.02	3165.78	-0.28424	197.93	0.04	5669.44	-0.59910	0.84552
29	197.81	0.09	3173.66	-0.28462	197.96	0.02	5668.33	-0.59912	0.84570
28	197.50	0.25	3184.44	-0.28513	197.95	0.03	5668.40	-0.59909	0.84556
27	197.09	0.46	3198.10	-0.28578	197.91	0.05	5668.50	-0.59895	0.84583
26	196.72	0.65	3210.66	-0.28638	197.91	0.05	5668.48	-0.59894	0.84502
25	196.30	0.86	3224.20	-0.28702	197.94	0.03	5668.01	-0.59901	0.84531
24	195.68	1.17	3244.40	-0.28799	197.91	0.04	5667.78	-0.59890	0.84493
23	194.93	1.55	3268.45	-0.28914	197.94	0.03	5667.73	-0.59899	0.84523
22	193.98	2.03	3298.29	-0.29058	197.97	0.01	5668.70	-0.59929	0.84597

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 39 WHICH HAS AN OBSERVED VALUE OF: 200.00

# CASES	REDUCED (LEARNING CURVE) MODEL				FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL				
USED	PREDICTION	% DEVIATION	EST B0	EST B1	PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
38	200.83	-0.42	3127.28	-0.28244	200.18	-0.09	5668.40	-0.59916	0.84582
37	200.83	-0.42	3127.32	-0.28244	200.17	-0.09	5668.67	-0.59913	0.84569
36	200.78	-0.39	3129.40	-0.28254	200.16	-0.08	5669.13	-0.59913	0.84565
35	200.67	-0.33	3133.44	-0.28273	200.15	-0.08	5669.31	-0.59918	0.84552
34	200.56	-0.28	3137.52	-0.28292	200.18	-0.09	5668.95	-0.59917	0.84581
33	200.37	-0.19	3144.40	-0.28324	200.16	-0.08	5669.46	-0.59914	0.84566
32	200.18	-0.09	3151.45	-0.28357	200.15	-0.08	5669.40	-0.59910	0.84553
31	199.99	0.01	3158.26	-0.28389	200.15	-0.07	5669.41	-0.59908	0.84543
30	199.77	0.11	3165.78	-0.28424	200.15	-0.08	5669.44	-0.59910	0.84552
29	199.55	0.23	3173.66	-0.28462	200.18	-0.09	5668.33	-0.59912	0.84570
28	199.23	0.38	3184.44	-0.28513	200.17	-0.09	5668.40	-0.59909	0.84556
27	198.83	0.59	3198.10	-0.28578	200.13	-0.07	5668.50	-0.59895	0.84583
26	198.45	0.77	3210.66	-0.28638	200.13	-0.07	5668.48	-0.59894	0.84502
25	198.04	0.98	3224.20	-0.28702	200.16	-0.08	5668.01	-0.59901	0.84531
24	197.41	1.29	3244.40	-0.28799	200.14	-0.07	5667.78	-0.59890	0.84493
23	196.66	1.67	3268.45	-0.28914	200.16	-0.08	5667.73	-0.59899	0.84523
22	195.71	2.14	3298.29	-0.29058	200.20	-0.10	5668.70	-0.59929	0.84597
21	194.39	2.83	3337.76	-0.29236	200.17	-0.08	5667.92	-0.59992	0.84537

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 38 WHICH HAS AN OBSERVED VALUE OF: 203.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
37	202.94	0.03	3127.32	-0.28244	202.82	0.09	5668.67	-0.59913	0.84569
36	202.89	0.05	3129.40	-0.28254	202.81	0.09	5669.13	-0.59913	0.84565
35	202.78	0.11	3133.44	-0.28273	202.80	0.10	5669.31	-0.59910	0.84552
34	202.67	0.16	3137.52	-0.28292	202.83	0.09	5668.95	-0.59917	0.84581
33	202.48	0.25	3144.40	-0.28324	202.81	0.09	5669.46	-0.59914	0.84566
32	202.29	0.35	3151.45	-0.28357	202.80	0.10	5669.40	-0.59910	0.84553
31	202.10	0.44	3158.26	-0.28389	202.80	0.10	5669.41	-0.59908	0.84543
30	201.89	0.55	3165.78	-0.28424	202.80	0.10	5669.44	-0.59910	0.84532
29	201.66	0.66	3173.66	-0.28462	202.83	0.08	5668.33	-0.59912	0.84570
28	201.35	0.81	3184.44	-0.28513	202.82	0.09	5668.40	-0.59909	0.84556
27	200.94	1.01	3198.10	-0.28578	202.78	0.11	5668.50	-0.59895	0.84503
26	200.57	1.20	3210.66	-0.28638	202.78	0.11	5668.48	-0.59894	0.84502
25	200.16	1.40	3224.20	-0.28792	202.81	0.09	5668.01	-0.59901	0.84531
24	199.53	1.71	3244.40	-0.28799	202.79	0.10	5667.78	-0.59890	0.84493
23	198.78	2.08	3268.45	-0.28914	202.81	0.09	5667.73	-0.59899	0.84528
22	197.83	2.55	3298.29	-0.29058	202.85	0.08	5668.70	-0.59920	0.84597
21	196.51	3.20	3339.76	-0.29256	202.82	0.09	5667.92	-0.59902	0.84537
20	194.78	4.05	3393.89	-0.29513	202.74	0.13	5664.33	-0.59840	0.84359

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 37 WHICH HAS AN OBSERVED VALUE OF: 206.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
36	205.06	0.45	3129.40	-0.28254	205.84	0.08	5669.13	-0.59913	0.84565
35	204.96	0.51	3133.44	-0.28273	205.83	0.08	5669.31	-0.59910	0.84552
34	204.85	0.56	3137.52	-0.28292	205.86	0.07	5668.95	-0.59917	0.84581
33	204.66	0.65	3144.40	-0.28324	205.84	0.08	5669.46	-0.59914	0.84566
32	204.47	0.74	3151.45	-0.28357	205.83	0.08	5669.40	-0.59910	0.84553
31	204.28	0.84	3158.26	-0.28389	205.83	0.08	5669.41	-0.59908	0.84543
30	204.06	0.94	3165.78	-0.28424	205.83	0.08	5669.44	-0.59910	0.84532
29	203.84	1.05	3173.66	-0.28462	205.86	0.07	5668.33	-0.59912	0.84570
28	203.53	1.20	3184.44	-0.28513	205.85	0.07	5668.40	-0.59909	0.84556
27	203.12	1.40	3198.10	-0.28578	205.81	0.09	5668.50	-0.59895	0.84503
26	202.75	1.58	3210.66	-0.28638	205.81	0.09	5668.48	-0.59894	0.84502
25	202.33	1.78	3224.20	-0.28702	205.84	0.08	5668.01	-0.59781	0.84531
24	201.71	2.08	3244.40	-0.28799	205.82	0.09	5667.78	-0.59890	0.84493
23	200.96	2.45	3268.45	-0.28914	205.81	0.08	5667.73	-0.59899	0.84528
22	200.01	2.91	3298.29	-0.29050	205.88	0.06	5668.70	-0.59920	0.84597
21	198.69	3.55	3339.76	-0.29256	205.85	0.08	5667.92	-0.59902	0.84537
20	196.96	4.39	3393.89	-0.29513	205.76	0.12	5664.33	-0.59840	0.84359
19	194.66	5.50	3466.00	-0.29853	205.77	0.11	5664.84	-0.59849	0.84366

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 36 WHICH HAS AN OBSERVED VALUE OF: 209.00

# Cases Used	Reduced (Learning Curve) Model: Prediction	% Deviation	Est B0	Est B1	Full (Cumulative Production & Production Rate) Model: Prediction	% Deviation	Est B0	Est B1	Est B2
35	207.22	0.85	3133.44	-0.28273	208.83	0.08	5669.31	-0.59910	0.84552
34	207.11	0.91	3137.52	-0.28292	208.86	0.07	5668.95	-0.59917	0.84581
33	206.92	0.99	3144.40	-0.28324	208.84	0.08	5669.46	-0.59914	0.84566
32	206.73	1.09	3151.45	-0.28357	208.83	0.08	5669.40	-0.59910	0.84553
31	206.54	1.18	3158.26	-0.28389	208.82	0.08	5669.41	-0.59908	0.84543
30	206.33	1.28	3165.78	-0.28424	208.83	0.08	5669.44	-0.59910	0.84552
29	206.10	1.39	3173.66	-0.28462	208.86	0.07	5668.33	-0.59912	0.84570
28	205.79	1.54	3184.44	-0.28513	208.85	0.07	5668.40	-0.59909	0.84556
27	205.39	1.73	3190.10	-0.28578	208.81	0.09	5668.50	-0.59895	0.84503
26	205.01	1.91	3210.66	-0.28638	208.81	0.09	5668.48	-0.59894	0.84502
25	204.60	2.11	3224.20	-0.28702	208.84	0.08	5668.61	-0.59901	0.84531
24	203.98	2.40	3244.40	-0.28799	208.82	0.09	5667.78	-0.59890	0.84493
23	203.22	2.76	3268.45	-0.28914	208.84	0.08	5667.73	-0.59899	0.84528
22	202.28	3.22	3298.29	-0.29058	208.88	0.06	5668.70	-0.59920	0.84597
21	200.96	3.85	3339.76	-0.29256	208.85	0.07	5667.92	-0.59902	0.84537
20	199.23	4.68	3393.89	-0.29513	208.76	0.12	5664.33	-0.59840	0.84339
19	196.93	5.78	3466.68	-0.29853	208.77	0.11	5664.84	-0.59849	0.84366
18	194.36	7.04	3549.33	-0.30240	208.67	0.16	5660.02	-0.59772	0.84127

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 35 WHICH HAS AN OBSERVED VALUE OF: 211.00

# Cases Used	Reduced (Learning Curve) Model: Prediction	% Deviation	Est B0	Est B1	Full (Cumulative Production & Production Rate) Model: Prediction	% Deviation	Est B0	Est B1	Est B2
34	209.24	0.83	3137.52	-0.28292	211.36	-0.17	5668.95	-0.59917	0.84581
33	209.06	0.92	3144.40	-0.28324	211.34	-0.16	5669.46	-0.59914	0.84566
32	208.86	1.01	3151.45	-0.28357	211.33	-0.16	5669.40	-0.59910	0.84553
31	208.67	1.10	3158.26	-0.28389	211.33	-0.15	5669.41	-0.59908	0.84543
30	208.46	1.20	3165.78	-0.28424	211.33	-0.16	5669.44	-0.59910	0.84552
29	208.24	1.31	3173.66	-0.28462	211.36	-0.17	5668.33	-0.59912	0.84570
28	207.93	1.46	3184.44	-0.28513	211.35	-0.17	5668.40	-0.59909	0.84556
27	207.52	1.65	3190.10	-0.28578	211.31	-0.15	5668.50	-0.59895	0.84503
26	207.15	1.82	3210.66	-0.28638	211.31	-0.15	5668.48	-0.59894	0.84502
25	206.74	2.02	3224.20	-0.28702	211.34	-0.16	5668.61	-0.59901	0.84531
24	206.12	2.31	3244.40	-0.28799	211.32	-0.15	5667.78	-0.59890	0.84493
23	205.37	2.67	3268.45	-0.28914	211.34	-0.16	5667.73	-0.59899	0.84528
22	204.42	3.12	3298.29	-0.29058	211.38	-0.18	5668.70	-0.59920	0.84597
21	203.10	3.74	3339.76	-0.29256	211.35	-0.16	5667.92	-0.59902	0.84537
20	201.37	4.56	3393.89	-0.29513	211.26	-0.12	5664.33	-0.59840	0.84339
19	199.07	5.65	3466.68	-0.29853	211.27	-0.13	5664.84	-0.59849	0.84366
18	196.44	6.90	3549.33	-0.30240	211.17	-0.08	5660.02	-0.59772	0.84127
17	193.50	8.29	3642.99	-0.30670	211.24	-0.11	5663.59	-0.59925	0.84293

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 34 WHICH HAS AN OBSERVED VALUE OF: 214.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
33	211.12	1.35	3144.40	-0.28324	213.74	0.12	5669.46	-0.59914	0.84566
32	210.92	1.44	3151.45	-0.28357	213.73	0.12	5669.40	-0.59910	0.84553
31	210.74	1.53	3158.26	-0.28389	213.73	0.13	5669.41	-0.59908	0.84543
30	210.53	1.62	3165.78	-0.28424	213.73	0.12	5669.44	-0.59910	0.84552
29	210.30	1.73	3173.66	-0.28462	213.76	0.11	5668.33	-0.59912	0.84579
28	209.99	1.87	3184.44	-0.28513	213.75	0.12	5668.40	-0.59909	0.84556
27	209.59	2.06	3198.10	-0.28578	213.71	0.13	5668.50	-0.59895	0.84503
26	209.21	2.24	3210.66	-0.28638	213.71	0.13	5668.48	-0.59894	0.84502
25	208.80	2.43	3224.20	-0.28702	213.74	0.12	5668.01	-0.59901	0.84531
24	208.18	2.72	3244.40	-0.28799	213.72	0.13	5667.78	-0.59690	0.84493
23	207.43	3.07	3268.45	-0.28914	213.75	0.12	5667.73	-0.59899	0.84528
22	206.49	3.51	3298.29	-0.29053	213.78	0.10	5668.70	-0.59920	0.84597
21	205.17	4.13	3339.76	-0.29256	213.75	0.12	5667.92	-0.59902	0.84537
20	203.44	4.94	3393.89	-0.29513	213.66	0.16	5664.33	-0.59840	0.84339
19	201.14	6.01	3466.08	-0.29853	213.67	0.15	5664.84	-0.59349	0.84366
18	198.50	7.24	3549.33	-0.30240	213.57	0.20	5660.82	-0.59772	0.84127
17	195.57	8.61	3642.99	-0.30670	213.64	0.17	5663.39	-0.59825	0.84293
16	192.19	10.19	3751.94	-0.31161	213.53	0.22	5657.92	-0.59740	0.84026

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 33 WHICH HAS AN OBSERVED VALUE OF: 216.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
32	213.13	1.33	3151.45	-0.28357	215.90	0.05	5669.40	-0.59910	0.84553
31	212.95	1.41	3158.26	-0.28389	215.89	0.05	5669.41	-0.59908	0.84543
30	212.74	1.51	3165.78	-0.28424	215.90	0.05	5669.44	-0.59910	0.84552
29	212.51	1.61	3173.66	-0.28462	215.93	0.03	5668.33	-0.59912	0.84570
28	212.20	1.76	3184.44	-0.28513	215.92	0.04	5668.40	-0.59909	0.84556
27	211.80	1.94	3198.10	-0.28578	215.88	0.06	5668.50	-0.59895	0.84503
26	211.43	2.12	3210.66	-0.28638	215.88	0.06	5668.48	-0.59894	0.84502
25	211.02	2.31	3224.20	-0.28702	215.91	0.04	5668.01	-0.59901	0.84531
24	210.40	2.59	3244.40	-0.28799	215.88	0.05	5667.78	-0.59690	0.84493
23	209.65	2.94	3268.45	-0.28914	215.91	0.04	5667.73	-0.59899	0.84528
22	208.71	3.33	3298.29	-0.29058	215.93	0.03	5668.70	-0.59920	0.84597
21	207.39	3.99	3339.76	-0.29256	215.91	0.04	5667.92	-0.59902	0.84537
20	205.66	4.79	3393.89	-0.29513	215.83	0.08	5664.33	-0.59840	0.84339
19	203.36	5.85	3466.08	-0.29853	215.84	0.08	5664.84	-0.59349	0.84366
18	200.72	7.07	3549.33	-0.30240	215.74	0.12	5660.82	-0.59772	0.84127
17	197.79	8.43	3642.99	-0.30670	215.81	0.09	5663.39	-0.59825	0.84293
16	194.41	10.00	3751.94	-0.31161	215.69	0.14	5657.92	-0.59740	0.84026
15	190.90	11.62	3866.30	-0.31669	215.74	0.12	5660.06	-0.59774	0.84132

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 32 WHICH HAS AN OBSERVED VALUE OF: 218.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL: PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL: PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
31	215.32	1.23	3158.26	-0.28389	217.91	0.04	5669.41	-0.59900	0.84543
30	215.11	1.33	3165.78	-0.28424	217.92	0.04	5669.44	-0.59910	0.84552
29	214.89	1.43	3173.66	-0.28462	217.95	0.02	5668.33	-0.59912	0.84570
28	214.58	1.57	3184.44	-0.28513	217.94	0.03	5668.40	-0.59909	0.84556
27	214.18	1.75	3198.10	-0.28578	217.90	0.05	5668.58	-0.59895	0.84503
26	213.81	1.92	3210.66	-0.28638	217.90	0.05	5668.43	-0.59894	0.84592
25	213.40	2.11	3224.20	-0.28702	217.93	0.03	5668.01	-0.59901	0.84531
24	212.78	2.40	3244.40	-0.28799	217.90	0.04	5667.78	-0.59870	0.84493
23	212.03	2.74	3260.45	-0.28914	217.93	0.03	5667.73	-0.59899	0.84528
22	211.09	3.17	3298.29	-0.29058	217.96	0.02	5668.70	-0.59920	0.84597
21	209.77	3.78	3339.76	-0.29256	217.93	0.03	5667.92	-0.59902	0.84537
20	208.04	4.57	3393.89	-0.29513	217.84	0.07	5664.33	-0.59840	0.84339
19	205.74	5.62	3466.08	-0.29853	217.86	0.07	5664.84	-0.59849	0.84366
18	203.11	6.83	3549.33	-0.30240	217.76	0.11	5660.02	-0.59772	0.84127
17	200.17	8.18	3642.99	-0.30670	217.83	0.08	5663.37	-0.59825	0.84273
16	196.70	9.73	3751.94	-0.31161	217.71	0.13	5657.92	-0.59749	0.84026
15	193.28	11.34	3866.38	-0.31669	217.76	0.11	5660.06	-0.59774	0.84132
14	189.80	12.93	3981.08	-0.32178	217.62	0.18	5654.38	-0.59678	0.83827

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 31 WHICH HAS AN OBSERVED VALUE OF: 221.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL: PREDICTION	% DEVIATION	EST B0	EST B1	FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL: PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
30	218.11	1.31	3165.78	-0.28424	221.08	-0.03	5669.44	-0.59910	0.84552
29	217.89	1.41	3173.66	-0.28462	221.11	-0.05	5668.33	-0.59912	0.84570
28	217.58	1.55	3184.44	-0.28513	221.10	-0.04	5668.40	-0.59909	0.84556
27	217.18	1.73	3198.10	-0.28578	221.06	-0.03	5668.58	-0.59895	0.84503
26	216.81	1.90	3210.66	-0.28639	221.06	-0.03	5668.48	-0.59894	0.84592
25	216.40	2.08	3224.28	-0.28702	221.09	-0.04	5668.01	-0.59901	0.84531
24	215.79	2.36	3244.40	-0.28799	221.06	-0.03	5667.78	-0.59870	0.84493
23	215.04	2.70	3260.45	-0.28914	221.09	-0.04	5667.73	-0.59899	0.84528
22	214.10	3.12	3298.29	-0.29058	221.12	-0.06	5668.70	-0.59920	0.84597
21	212.78	3.72	3339.76	-0.29256	221.09	-0.04	5667.92	-0.59902	0.84537
20	211.06	4.50	3393.89	-0.29513	221.00	-0.00	5664.33	-0.59840	0.84339
19	208.76	5.54	3466.08	-0.29853	221.02	-0.01	5664.84	-0.59849	0.84366
18	206.12	6.73	3549.33	-0.30240	220.92	0.04	5660.02	-0.59772	0.84127
17	203.18	8.06	3642.99	-0.30670	220.99	0.01	5663.37	-0.59825	0.84273
16	199.80	9.59	3751.94	-0.31161	220.87	0.06	5657.92	-0.59748	0.84026
15	196.28	11.18	3866.38	-0.31669	220.92	0.04	5660.06	-0.59774	0.84132
14	192.80	12.76	3981.08	-0.32178	220.77	0.10	5654.38	-0.59678	0.83827
13	187.66	14.13	4095.78	-0.32620	220.85	0.07	5656.45	-0.59718	0.83959

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 30 WHICH HAS AN OBSERVED VALUE OF: 224.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL				FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL				
	PREDICTION	% DEVIATION	EST B0	EST B1	PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
29	221.04	1.32	3173.66	-0.28462	224.37	-0.17	5668.33	-0.59912	0.84570
28	220.74	1.46	3184.44	-0.28513	224.36	-0.16	5668.43	-0.59909	0.84556
27	220.34	1.63	3198.10	-0.28578	224.32	-0.14	5668.50	-0.59895	0.84583
26	219.97	1.80	3210.66	-0.28638	224.32	-0.14	5668.48	-0.59894	0.84502
25	219.56	1.98	3224.20	-0.28702	224.35	-0.16	5668.01	-0.59901	0.84531
24	218.95	2.26	3244.40	-0.28799	224.33	-0.15	5667.78	-0.59870	0.84493
23	218.20	2.59	3268.45	-0.28914	224.35	-0.16	5667.73	-0.59899	0.84528
22	217.26	3.01	3298.29	-0.29058	224.39	-0.17	5668.70	-0.59920	0.84577
21	215.95	3.59	3339.76	-0.29256	224.36	-0.16	5667.92	-0.59902	0.84537
20	214.23	4.36	3393.89	-0.29513	224.27	-0.12	5664.33	-0.59840	0.84339
19	211.93	5.39	3466.00	-0.29853	224.28	-0.13	5664.84	-0.59849	0.84366
18	209.30	6.56	3549.33	-0.30240	224.18	-0.08	5660.02	-0.59772	0.84127
17	206.36	7.88	3642.99	-0.30670	224.25	-0.11	5663.39	-0.59825	0.84293
16	202.97	9.39	3751.94	-0.31161	224.14	-0.06	5657.92	-0.59740	0.84026
15	199.45	10.96	3866.30	-0.31669	224.18	-0.08	5660.06	-0.59774	0.84132
14	195.96	12.52	3981.00	-0.32170	224.04	-0.02	5654.38	-0.59678	0.83827
13	192.81	13.92	4095.78	-0.32620	224.11	-0.05	5656.45	-0.59718	0.83958
12	190.79	14.82	4152.42	-0.32906	224.45	-0.20	5663.51	-0.59878	0.84499

SHORTRANGE PREDICTIVE ABILITY COMPARISON

THE DATA PRESENTED BELOW IS FOR CASE # 29 WHICH HAS AN OBSERVED VALUE OF: 228.00

# CASES USED	REDUCED (LEARNING CURVE) MODEL				FULL (CUMULATIVE PRODUCTION & PRODUCTION RATE) MODEL				
	PREDICTION	% DEVIATION	EST B0	EST B1	PREDICTION	% DEVIATION	EST B0	EST B1	EST B2
28	224.06	1.73	3184.44	-0.28513	227.87	0.06	5668.40	-0.59909	0.84556
27	223.67	1.90	3198.10	-0.28578	227.84	0.07	5668.50	-0.59895	0.84563
26	223.30	2.06	3210.66	-0.28638	227.84	0.07	5668.48	-0.59894	0.84502
25	222.89	2.24	3224.20	-0.28702	227.86	0.06	5668.01	-0.59901	0.84531
24	222.28	2.51	3244.40	-0.28799	227.84	0.07	5667.78	-0.59870	0.84493
23	221.54	2.84	3268.45	-0.28914	227.87	0.06	5667.73	-0.59899	0.84528
22	220.60	3.25	3298.29	-0.29058	227.90	0.04	5668.70	-0.59920	0.84597
21	219.29	3.82	3339.76	-0.29256	227.87	0.06	5667.92	-0.59902	0.84537
20	217.57	4.58	3393.89	-0.29513	227.78	0.10	5664.33	-0.59840	0.84339
19	215.27	5.58	3466.00	-0.29853	227.79	0.09	5664.84	-0.59849	0.84366
18	212.64	6.74	3549.33	-0.30240	227.69	0.13	5660.02	-0.59772	0.84127
17	209.70	8.03	3642.99	-0.30670	227.76	0.10	5663.39	-0.59825	0.84293
16	206.31	9.51	3751.94	-0.31161	227.65	0.15	5657.92	-0.59740	0.84026
15	202.79	11.06	3866.30	-0.31669	227.70	0.13	5660.06	-0.59774	0.84132
14	199.29	12.59	3981.00	-0.32170	227.55	0.20	5654.38	-0.59678	0.83827
13	196.14	13.97	4095.78	-0.32620	227.62	0.17	5656.45	-0.59718	0.83958
12	194.11	14.86	4152.42	-0.32906	227.96	0.02	5663.51	-0.59878	0.84499
11	194.05	14.89	4154.60	-0.32915	227.88	0.05	5666.63	-0.59883	0.84479

SUMMARY OF PREDICTIVE ABILITY TESTS RESULTS		
ITEMS OF INTEREST	REDUCED MODEL	FULL MODEL
AVERAGE ABSOLUTE DEVIATION	11.66	0.21
VARIANCE OF ABSOLUTE DEVIATIONS	115.45	0.02
TEST STATISTIC (SEE NOTE)	---	19.19
TOTAL NUMBER OF TEST SITUATIONS	324	324
NUMBER OF PREDICTIONS WITHIN 5%	204	324
PERCENT OF PREDICTIONS WITHIN 5%	62.	100.
NUMBER OF PREDICTIONS WITHIN 10%	260	324
PERCENT OF PREDICTIONS WITHIN 10%	80.	100.

NOTE: IN TESTING FOR STATISTICAL SIGNIFICANCE USE STUDENT'S T DISTRIBUTION IF THE NUMBER OF TEST SITUATIONS ARE LESS THAN 60; OTHERWISE USE STANDARD NORMAL DISTRIBUTION. IN EITHER CASE THIS IS A ONE TAILED TEST. IF THE TEST STATISTIC IS GREATER THAN THE CRITICAL STATISTIC ONE MAY CONCLUDE THAT THE AVERAGE ABSOLUTE DEVIATION OBTAINED WITH THE FULL MODEL IS SIGNIFICANTLY LESS THAN THAT OBTAINED WITH THE REDUCED MODEL.

PROJECTION AND SENSITIVITY MATRIX

PROJECTED CUMULATIVE UNITS	PROJECTED PRODUCTION RATES								
	13.26	15.15	17.05	18.94	20.83	22.73	24.62	26.52	28.41
17343	145.5	162.9	180.0	196.8	213.3	229.6	245.7	261.6	277.3
17515	144.7	162.0	178.9	195.5	212.0	228.2	244.2	260.0	275.7
17687	143.8	161.0	177.9	194.3	210.8	226.9	242.8	258.5	274.1
17858	143.0	160.1	176.9	193.4	209.6	225.6	241.4	257.0	272.5
18030	142.2	159.2	175.9	192.3	208.4	224.3	240.0	255.6	270.9
18202	141.4	158.3	174.9	191.2	207.2	223.0	238.7	254.1	269.4
18374	140.6	157.4	173.9	190.1	206.0	221.8	237.3	252.7	267.9
18545	139.8	156.5	172.9	189.0	204.9	220.6	236.0	251.3	266.4
18717	139.0	155.6	172.0	188.0	203.8	219.3	234.7	249.9	264.9
18889	138.3	154.8	171.0	187.0	202.7	218.1	233.4	248.5	263.5
19060	137.5	154.0	170.1	186.0	201.6	217.0	232.2	247.2	262.1
19232	136.8	153.1	169.2	185.0	200.5	215.8	230.9	245.9	260.6
19404	136.1	152.3	168.3	184.0	199.4	214.7	229.7	244.6	259.3
19576	135.3	151.5	167.4	183.0	198.4	213.5	228.5	243.3	257.9
19747	134.6	150.7	166.5	182.1	197.3	212.4	227.3	242.0	256.5
19919	133.9	150.0	165.7	181.1	196.3	211.3	226.1	240.7	255.2
20091	133.2	149.2	164.8	180.2	195.3	210.2	225.0	239.5	253.9
20262	132.6	148.4	164.0	179.3	194.3	209.2	223.8	238.3	252.6
20434	131.9	147.7	163.1	178.4	193.3	208.1	222.7	237.1	251.3
20606	131.2	146.9	162.3	177.5	192.4	207.1	221.6	235.9	250.1
20778	130.6	146.2	161.5	176.6	191.4	206.0	220.5	234.7	248.8
20949	129.9	145.5	160.7	175.7	190.5	205.0	219.4	233.6	247.6
21121	129.3	144.8	159.9	174.9	189.5	204.0	218.3	232.4	246.4
21293	128.7	144.1	159.2	174.0	188.6	203.0	217.3	231.3	245.2
21464	128.1	143.4	158.4	173.2	187.7	202.1	216.2	230.2	244.0
21636	127.5	142.7	157.7	172.4	186.8	201.1	215.2	229.1	242.9
21808	126.9	142.0	156.9	171.5	185.9	200.1	214.2	228.0	241.7
21980	126.3	141.4	156.2	170.7	185.1	199.2	213.2	227.0	240.6
22151	125.7	140.7	155.4	169.9	184.2	198.3	212.2	225.9	239.5
22323	125.1	140.1	154.7	169.2	183.4	197.4	211.2	224.9	238.4
22495	124.5	139.4	154.0	168.4	182.5	196.5	210.2	223.8	237.3
22667	124.0	138.8	153.3	167.6	181.7	195.6	209.3	222.8	236.2
22838	123.4	138.2	152.6	166.9	180.9	194.7	208.3	221.8	235.1
23010	122.8	137.5	151.9	166.1	180.1	193.8	207.4	220.8	234.1
23182	122.3	136.9	151.3	165.4	179.3	192.9	206.5	219.8	233.0
23353	121.8	136.3	150.6	164.6	178.5	192.1	205.6	218.9	232.0
23525	121.2	135.7	149.9	163.9	177.7	191.3	204.7	217.9	231.0
23697	120.7	135.1	149.3	163.2	176.9	190.4	203.8	217.0	230.0
23869	120.2	134.5	148.6	162.5	176.1	189.6	202.9	216.0	229.0
24040	119.7	134.0	148.0	161.8	175.4	188.8	202.0	215.1	228.0
24212	119.1	133.4	147.4	161.1	174.6	188.0	201.2	214.2	227.0
24384	118.6	132.8	146.8	160.4	173.9	187.2	200.3	213.3	226.1
24555	118.1	132.3	146.1	159.8	173.2	186.4	199.5	212.4	225.1
24727	117.7	131.7	145.5	159.1	172.5	185.6	198.6	211.5	224.2
24899	117.2	131.2	144.9	158.4	171.7	184.9	197.8	210.6	223.3
25071	116.7	130.6	144.3	157.8	171.0	184.1	197.0	209.7	222.4
25242	116.2	130.1	143.7	157.1	170.3	183.4	196.2	208.9	221.5
25414	115.7	129.6	143.2	156.5	169.6	182.6	195.4	208.0	220.6
25586	115.3	129.1	142.6	155.9	169.0	181.9	194.6	207.2	219.7
25757	114.8	128.5	142.0	155.3	168.3	181.1	193.8	206.4	218.8

NOTE: 1. PROJECTED VALUES FOR DIRECT LABOR HOURS MAY BE READ FROM THE ABOVE MATRIX BY MATCHING A GIVEN PRODUCTION RATE WITH A GIVEN NUMBER OF CUMULATIVE UNITS AND READING THE VALUE FOR DIRECT LABOR HOURS FOUND AT THE INTERSECTION OF THE CORESPONDING ROW AND COLUMN. FORECASTING MODEL IS THE CUMULATIVE PRODUCTION & PRODUCTION RATE MODEL.

2. PROJECTION INTERVAL FOR CUMULATIVE UNITS IS 1% OF THE LAST OBSERVED VALUE OF CUMULATIVE UNITS.

3. PROJECTION VALUES FOR PRODUCTION RATE ARE 70, 80, 90, 100, 110, 120, 130, 140, AND 150 PERCENT OF THE LAST OBSERVED VALUE OF PRODUCTION RATE.

PROJECTION AND SENSITIVITY MATRIX

PROJECTED CUMULATIVE UNITS	PROJECTED PRODUCTION RATES								
	13.26	15.15	17.05	18.94	20.83	22.73	24.62	26.52	28.41
25929	114.4	128.0	141.4	154.6	167.6	180.4	193.1	205.6	217.9
26101	113.9	127.5	140.9	154.0	167.0	179.7	192.3	204.7	217.1
26273	113.5	127.0	140.3	153.4	166.3	179.0	191.5	203.9	216.2
26444	113.0	126.5	139.8	152.8	165.7	178.3	190.8	203.2	215.4
26616	112.6	126.0	139.2	152.2	165.0	177.6	190.1	202.4	214.5
26788	112.1	125.6	138.7	151.6	164.4	176.9	189.3	201.6	213.7
26960	111.7	125.1	138.2	151.1	163.8	176.3	188.6	200.8	212.9
27131	111.3	124.6	137.7	150.5	163.1	175.6	187.9	200.1	212.1
27303	110.9	124.1	137.1	149.9	162.5	174.9	187.2	199.3	211.3
27475	110.5	123.7	136.6	149.4	161.9	174.3	186.5	198.5	210.5
27646	110.0	123.2	136.1	148.8	161.3	173.6	185.8	197.8	209.7
27818	109.6	122.8	135.6	148.3	160.7	173.0	185.1	197.1	208.9
27990	109.2	122.3	135.1	147.7	160.1	172.3	184.4	196.4	208.2
28162	108.8	121.8	134.6	147.2	159.5	171.7	183.7	195.6	207.4
28333	108.4	121.4	134.1	146.6	158.9	171.1	183.1	194.9	206.6
28505	108.0	121.0	133.6	146.1	158.4	170.5	182.4	194.2	205.9
28677	107.7	120.5	133.2	145.6	157.8	169.9	181.8	193.5	205.2
28848	107.3	120.1	132.7	145.1	157.2	169.3	181.1	192.8	204.4
29020	106.9	119.7	132.2	144.5	156.7	168.7	180.5	192.1	203.7
29192	106.5	119.3	131.8	144.0	156.1	168.1	179.8	191.5	203.0
29364	106.1	118.8	131.3	143.5	155.6	167.5	179.2	190.8	202.3
29535	105.8	118.4	130.8	143.0	155.0	166.9	178.6	190.1	201.6
29707	105.4	118.0	130.4	142.5	154.5	166.3	178.0	189.5	200.9
29879	105.0	117.6	129.9	142.0	154.0	165.7	177.3	188.8	200.2
30050	104.7	117.2	129.5	141.6	153.4	165.2	176.7	188.2	199.5
30222	104.3	116.8	129.0	141.1	152.9	164.6	176.1	187.5	198.8
30394	104.0	116.4	128.6	140.6	152.4	164.0	175.5	186.9	198.1
30566	103.6	116.0	128.2	140.1	151.9	163.5	174.9	186.3	197.5
30737	103.3	115.6	127.7	139.7	151.4	162.9	174.4	185.6	196.8
30909	102.9	115.2	127.3	139.2	150.9	162.4	173.8	185.0	196.1
31081	102.6	114.9	126.9	138.7	150.4	161.9	173.2	184.4	195.5
31252	102.2	114.5	126.5	138.3	149.9	161.3	172.6	183.8	194.8
31424	101.9	114.1	126.1	137.8	149.4	160.8	172.1	183.2	194.2
31596	101.6	113.7	125.6	137.4	148.9	160.3	171.5	182.6	193.6
31768	101.3	113.4	125.2	136.9	148.4	159.8	170.9	182.0	192.9
31939	100.9	113.0	124.8	136.5	147.9	159.2	170.4	181.4	192.3
32111	100.6	112.6	124.4	136.0	147.5	158.7	169.8	180.8	191.7
32283	100.3	112.3	124.0	135.6	147.0	158.2	169.3	180.3	191.1
32455	100.0	111.9	123.6	135.2	146.5	157.7	168.8	179.7	190.5
32626	99.6	111.6	123.3	134.7	146.1	157.2	168.2	179.1	189.9
32798	99.3	111.2	122.9	134.3	145.6	156.7	167.7	178.6	189.3
32970	99.0	110.9	122.5	133.9	145.1	156.2	167.2	178.0	188.7
33141	98.7	110.5	122.1	133.5	144.7	155.8	166.7	177.4	188.1
33313	98.4	110.2	121.7	133.1	144.2	155.3	166.1	176.9	187.5
33485	98.1	109.8	121.4	132.7	143.8	154.8	165.6	176.4	187.0
33657	97.8	109.5	121.0	132.3	143.4	154.3	165.1	175.8	186.4
33828	97.5	109.2	120.6	131.9	142.9	153.8	164.6	175.3	185.8
34000	97.2	108.8	120.2	131.5	142.5	153.4	164.1	174.7	185.3
34172	96.9	108.5	119.9	131.1	142.1	152.9	163.6	174.2	184.7
34343	96.6	108.2	119.5	130.7	141.6	152.5	163.1	173.7	184.1

NOTE: 1. PROJECTED VALUES FOR DIRECT LABOR HOURS MAY BE READ FROM THE ABOVE MATRIX BY MATCHING A GIVEN PRODUCTION RATE WITH A GIVEN NUMBER OF CUMULATIVE UNITS AND READING THE VALUE FOR DIRECT LABOR HOURS FOUND AT THE INTERSECTION OF THE CORRESPONDING ROW AND COLUMN. FORECASTING MODEL IS THE CUMULATIVE PRODUCTION & PRODUCTION RATE MODEL.

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APPENDIX B

MULTICOLLINEARITY

The purpose of this appendix is to pull together some observations and references about multicollinearity. The information is separately recorded from Chapter III to expand the necessary treatment of multicollinearity without burdening the reader with unnecessary detail. The comments assume familiarity with multiple linear regression as a statistical tool, and should be read in conjunction with the comments about multicollinearity in Chapter III.

Multicollinearity is defined (18:340) as the condition existing in multiple regression analysis when two or more of the independent variables are highly intercorrelated. It was noted in Chapter III that one of the assumptions of this research was that the varying degrees of multicollinearity did not importantly impair the models' short-range predictive ability. Other notable references are as follow:

1. Congleton and Kinton (8:33) and Smith (21:46) assumed that multicollinearity did not impair predictive ability.

2. Spurr and Bonini (22:612) reaffirmed the unreliability of a regression coefficient under conditions of multicollinearity, as cited in Chapter III under a different reference (17:341). Spurr and Bonini also stated

that multicollinearity may not affect the model's ability to predict (22:612). Put another way, the standard error of the estimate (which is essentially the standard deviation of the residuals) may well be the same as if there were no multicollinearity. This can hold true because the sampling errors of each coefficient (in a model with two independent variables) tend to cancel each other (22:611).

As noted in Chapter III, multicollinearity is likely to be present in business and economic data (17:255). It was present in this research, but as in the usual case (17:345) it created no problems in predictive ability. But it could have. In fact, multicollinearity can pose problems so severe that reparameterization or omission of a variable becomes necessary to increase computational accuracy (17:346-351). Moreover, had the scope of the research been more extensive (e.g., less emphasis on predictive ability and more on assessment of individual independent variables), multicollinearity could have posed formidable problems.

Finally, a thought-provoking phenomenon which occasionally occurs in regression analysis in the presence of multicollinearity is the change of sign in the regression coefficient. For example, if B_1 were cumulative output and B_2 were production rate, a hypothetical result might be as follows. Assume B_2 entered stepwise

regression first (being forced) and showed a bivariate coefficient of -0.5. Further assume B_1 was then entered in the regression, and the B_2 partial coefficient became +0.5. The question of why this change occurred must arise.

The change-of-sign phenomenon was reported by Smith (21:134) in association with his analysis of the KC-135A data. Smith (21:142-144) attributed this phenomenon in the B_2 coefficient to the combined effects of highly correlated independent variables and the overpowering strength of the other (X_1) independent variable.

Spurr and Bonini (22:611) described the change in sign phenomenon as a peculiar result of multicollinearity. Moreover, the peculiar result and the reason for it, as discussed by Smith and supported by Spurr and Bonini, was corroborated by Nie and others (18:330) as follows:

> It is worth noting at this point that the relative magnitude of the partial regression coefficient of an independent variable can be quite different from its bivariate regression coefficient with the dependent variable since the bivariate coefficient is confounded with the effects of other correlated independent variables.

At least one example of the change-of-sign phenomenon occurred in this research. During data analysis in that instance the B_2 coefficient when the production rate variable entered regression first was -0.583 as reflected in Figure 3. Subsequently, following addition of the cumulative output variable to the regression, the B_2 coefficient

```
*********************************************************************
THE EQUATION FOR THIS MODEL IS:        YHAT = B0 * X2 ** B2
IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B2 * LOG(X2)
WHERE: LOG(B0) =  MASKED    STD ERROR = 0.26945    B0 =  MASKED
          B2 =-0.53306    STD ERROR = 0.04402
SUMMARY STATISTICS:
R SQUARED LOG   =0.80151            STD ERROR EST =     0.0598
MSE             =  0.00357          MSR           =  0.53384
F RATIO         = 149.4110          D. F. (N/D)   = 1/ 37
R SQUARED ACTUAL=0.77464
*********************************************************************
```

Fig. 3. Summary of Results of Regression on Production Rate Variable Alone

NOTE: Masked denotes proprietary data.

became +0.289 as is observed in Figure 4. This algebraic change of more than 0.8 resulted from the addition of the cumulative output variable to the regression in which the production rate variable was already present. The change in magnitude and sign of the coefficient were attributed to the multicollinearity between independent variables and, for the model which was being investigated, the over-powering strength of the cumulative output variable. The multicollinearity present in this particular analysis is evidenced in Figure 5 where it is observed that the correlation of cumulative output and production rate is greater than 0.97.

The following summary statements are offered, therefore, as support of prior investigation and existing theory.

1. The occurrence of multicollinearity is not uncommon in business and economic data. In this reasearch multicollinearity was evidenced by the high correlation

```
**********************************************************************
THE EQUATION FOR THIS MODEL IS:       YHAT = B0 * X1 ** B1 * X2 **B2
IN LOG FORM THIS MODEL BECOMES: LOG(YHAT) = LOG(B0) + B1 * LOG(X1) + B2 * LOG(X2)
WHERE: LOG(B0) =  MASKED    STD ERROR = 0.04958    B0 =  MASKED
           B1 =  MASKED    STD ERROR = 0.05789    F* =   32.7227
           B2 = 0.23930    STD ERROR = 0.14818    F* =    3.8091
SUMMARY STATISTICS:
R SQUARED LOG    =0.89602           STD ERROR EST =     0.0439
MSE              =  0.00192         MSR           =  0.29839
F RATIO          = 155.1170         D. F. (N/D)   = 2/ 36
R SQUARED ACTUAL=0.82789
**********************************************************************
```

Fig. 4. Summary Results of Regression on Production Rate and Cumulative Output

NOTE: Masked denotes proprietary data.

PEARSON CORRELATION COEFFICIENTS MATRIX

	Y	X1	X2
Y	1.0000000	-0.9407704	-0.8952799
X1	-0.9407704	1.0000000	0.9759556
X2	-0.8952799	0.9759556	1.0000000

Fig. 5. Pearson Correlation Coefficients Matrix

NOTE: Masked denotes proprietary data.

between the production rate, the variable of primary research interest, and the cumulative output.

2. While multicollinearity does affect the reliability of partial regression coefficients, it does not necessarily affect the model's ability to explain variation in the dependent variable.

3. Multicollinearity can be enough or a burden to cause remedial measures such as omission of a variable or reparameterization to improve computational accuracy. The probability of this burden increases if the purpose of the regression is other than prediction, i.e., description, control, or explanation.

4. The "peculiar" change-of-sign phenomenon is not rare; it is attributable to the synergism resulting from correlation of independent variables and their different (perhaps grossly different) strengths.

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BIOGRAPHICAL SKETCHES

BIOGRAPHICAL SKETCHES

Captain David Y. Stevens graduated from Kent State University in March 1970 with a Bachelor of Science Degree in Aerospace Technology. Commissioned through ROTC, his assignments included tours as SAC Missile Launch Officer and Wing Command Post Controller; in 1974 he was assigned as an ADCOM Missile Warning Detachment Operations Officer. Prior to AFIT he was on the first operations team activating the Cobra Dane Radar at Shemya AFB, AK. Upon graduation from the School of Systems and Logistics he will be assigned to the Contracting Branch, Electronics System Division of AFSC at Hanscom Field, MA.

Captain Jimmie Thomerson graduated from the Bloomington Campus School of Business, Indiana University in December 1973 with a Bachelor of Science Degree in Management. Commissioned through the Officer Training School in April 1974, he was an Executive Support Officer in SAC and AFCS before attending the School of Systems and Logistics. From 1965 until 1974 he was enlisted in the weather career field and served with MAC, SAC, and the MAAG in Thailand. Upon graduation from the School of Systems and Logistics he will be assigned to the San Antonio ALC as a Contracting Officer.